A Level Maths Transition Pack

Dear prospective students,

Please complete the weekly tasks to prepare you for the transition to A Level Maths. There are examples at the start of each exercise to help you, and solutions are provided (at the end of the booklet) so you can self-assess your work.

These tasks are the foundation of A Level Maths and securing these will help you enhance your mathematical skills and knowledge.

I have included Hegarty maths clips in case you would like these for additional support but completing these is not required.

You must complete and submit the weekly assessment at the end of each week. Solutions are not provided for these, and we will mark them to assess your understanding.

In terms of the course outline, you'll be studying Pure Maths, Statistics and Mechanics and you will sit three exams at the end of year 13.
Paper 1: Pure Mathematics (2 hours)
Paper 2: Pure Mathematics (2 hours)
Paper 3: Statistics & Mechanics (2 hours)

Thank you for choosing A Level Mathematics at Lambeth Academy

If you have any further questions, please do not hesitate to contact me.

Kind regards,
Mr Moodliar
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Year 11 Transition Tasks to A Level Mathematics

Week 1 Tasks

1. Indices HM: 102

Week 1 Tasks

2. Laws of Indices HM: 105, 106, 107

3. Negative Indices HM: 104, 105, 106, 107

4. Fractional Indices HM: 108, 109

5. Multiplying & Dividing Surds HM: 113, 114

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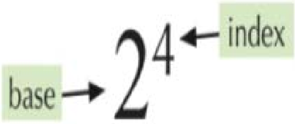
Indices

Week 1 Task 1

HM: 102, 105, 106, 107

Index notation (i.e. powers) can be used to show **repeated multiplication** of a number or letter. For example, $2 \times 2 \times 2 \times 2 = 2^4$. This is read as "2 **to the power** 4".

In index notation, the **base** is the value that you're multiplying (here, 2) and the **index** is the number of instances of that value (here, 4).



Indices

Week 1 Task 1

HM: 102, 105, 106, 107

Example 1

Rewrite the following using index notation: **a)** $3 \times 3 \times 3 \times 3 \times 3$ **b)** $b \times b \times b \times b \times c \times c$

a) There are five lots of 3. $3 \times 3 \times 3 \times 3 \times 3 = \mathbf{3^5}$

b) There are four *b*'s and two *c*'s. $b \times b \times b \times b \times c \times c = b^4 \times c^2 = \mathbf{b^4c^2}$

Indices

Week 1 Task 1

HM: 102, 105, 106, 107

Example 2

Simplify $\left(\frac{2}{5}\right)^2$.

With powers of fractions, the power is applied to both the top and bottom of the fraction.

$$\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \mathbf{\frac{4}{25}}$$

Indices

Week 1 Task

HM: 102, 105, 106, 107

Exercise 1

Q1 Using index notation, simplify the following.

a) $2 \times 2 \times 2 \times 2 \times 2$

b) $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

c) $3 \times 3 \times x \times x \times x \times y \times y$

Q2 Rewrite the following as powers of 10.

a) $10 \times 10 \times 10$

b) 10 million

c) 100 000 000

Q3 Using a calculator, evaluate the following.

a) 3^4

b) 2^8

c) 3^{10}

d) 9^5

e) 3×2^8

f) $8 + 2^5$

g) $8^7 \div 4^6$

h) $(9^3 + 4)^2$

Q4 Write the following as fractions without indices.

a) $\left(\frac{1}{2}\right)^2$

b) $\left(\frac{1}{2}\right)^3$

c) $\left(\frac{1}{4}\right)^2$

d) $\left(\frac{2}{3}\right)^2$

e) $\left(\frac{3}{10}\right)^2$

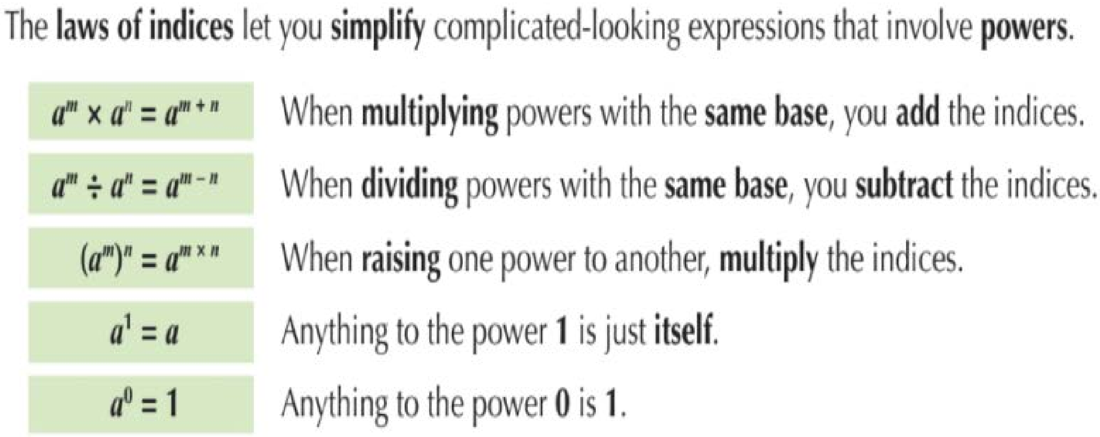
f) $\left(\frac{3}{2}\right)^3$

g) $\left(\frac{5}{3}\right)^4$

h) $\left(\frac{4}{3}\right)^3$

Laws of Indices

Week 1 Task 2
HM: 102, 105, 106, 107



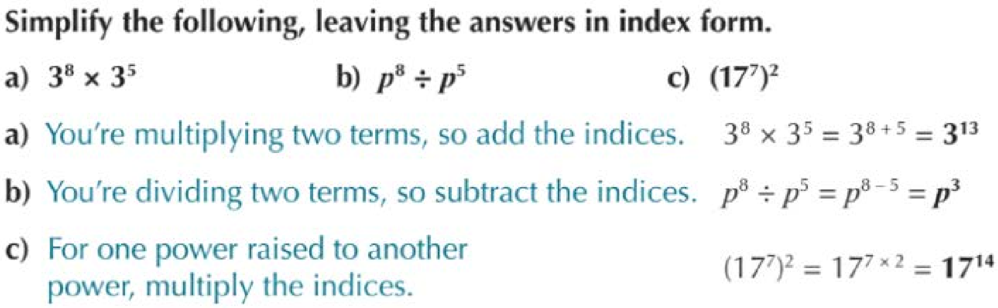
The **laws of indices** let you **simplify** complicated-looking expressions that involve **powers**.

$a^m \times a^n = a^{m+n}$	When **multiplying** powers with the **same base**, you **add** the indices.
$a^m \div a^n = a^{m-n}$	When **dividing** powers with the **same base**, you **subtract** the indices.
$(a^m)^n = a^{m \times n}$	When **raising** one power to another, **multiply** the indices.
$a^1 = a$	Anything to the power **1** is just **itself**.
$a^0 = 1$	Anything to the power **0** is **1**.

Laws of Indices

Week 1 Task 2
HM: 102, 105, 106, 107

Example 3



Simplify the following, leaving the answers in index form.

a) $3^8 \times 3^5$ **b)** $p^8 \div p^5$ **c)** $(17^7)^2$

a) You're multiplying two terms, so add the indices. $3^8 \times 3^5 = 3^{8+5} = \mathbf{3^{13}}$

b) You're dividing two terms, so subtract the indices. $p^8 \div p^5 = p^{8-5} = \boldsymbol{p^3}$

c) For one power raised to another power, multiply the indices. $(17^7)^2 = 17^{7 \times 2} = \mathbf{17^{14}}$

Laws of Indices

Week 1 Task 2

HM: 102, 105, 106, 107

Exercise 2

Q1 Simplify the following, leaving your answers in index form.

a) $3^2 \times 3^6$ b) $10^7 \div 10^3$ c) $a^6 \times a^4$ d) $(4^3)^3$

e) $8^6 \div 8^1$ f) 7×7^6 g) $(c^5)^4$ h) $\frac{b^8}{b^5}$

i) $f^{75} \div f^0$ j) $\frac{20^{228}}{20^{210}}$ k) $(g^{11})^8$ l) $(14^7)^d$

Q2 For each of the following, find the number that should replace the square.

a) $q^8 \div q^3 = q^{\blacksquare}$ b) $8^{\blacksquare} \times 8^{10} = 8^{12}$ c) $(6^{10})^4 = 6^{\blacksquare}$ d) $(15^6)^{\blacksquare} = 15^{24}$

e) $(9^{\blacksquare})^{10} = 9^{30}$ f) $r^7 \times r^{\blacksquare} = r^{13}$ g) $5^{\blacksquare} \div 5^6 = 5^7$ h) $12^{14} \div 12^{\blacksquare} = 12^7$

Q3 Simplify each expression. Leave your answers in index form.

a) $3^2 \times 3^5 \times 3^7$ b) $5^4 \times 5 \times 5^8$ c) $(p^6)^2 \times p^5$ d) $(9^4 \times 9^3)^5$

e) $7^3 \times 7^5 \div 7^6$ f) $8^3 \div 8^9 \times 8^7$ g) $(12^8 \div 12^4)^3$ h) $(q^3)^6 \div q^4$

Q4 Simplify each expression. Leave your answers in index form.

a) $\frac{3^4 \times 3^5}{3^6}$ b) $\frac{s^8 \times s^4}{s^3 \times s^6}$ c) $\left(\frac{6^3 \times 6^9}{6^7}\right)^3$ d) $\frac{2^5 \times 2^5}{(2^3)^2}$

e) $\frac{5^5 \times 5^5}{5^8 \div 5^3}$ f) $\frac{10^8 \div 10^3}{10^4 \div 10^4}$ g) $\frac{(t^6 \div t^3)^4}{t^9 \div t^4}$ h) $\frac{(8^5)^7 \div 8^{12}}{8^6 \times 8^{10}}$

Q5 a) Write: (i) 4 as a power of 2 (ii) 4^5 as a power of 2 (iii) $2^3 \times 4^5$ as a power of 2

b) Write: (i) 9×3^3 as a power of 3 (ii) $5 \times 25 \times 125$ as a power of 5 (iii) 16×2^6 as a power of 4

Negative Indices

Week 1 Task 3

HM: 104, 150, 106, 107

You can evaluate powers that have a **negative index** by taking the **reciprocal** of the base (i.e. turning it upside down — the reciprocal of a is $\frac{1}{a}$ and the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$) and making the index **positive**.

$$a^{-m} = \frac{1}{a^m}$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$$

Negative Indices

Week 1 Task

HM: 104, 105, 106, 107

Example 4

Evaluate 5^{-3}

1. Take the reciprocal of the base and make the index positive.
2. Evaluate the index in the denominator.

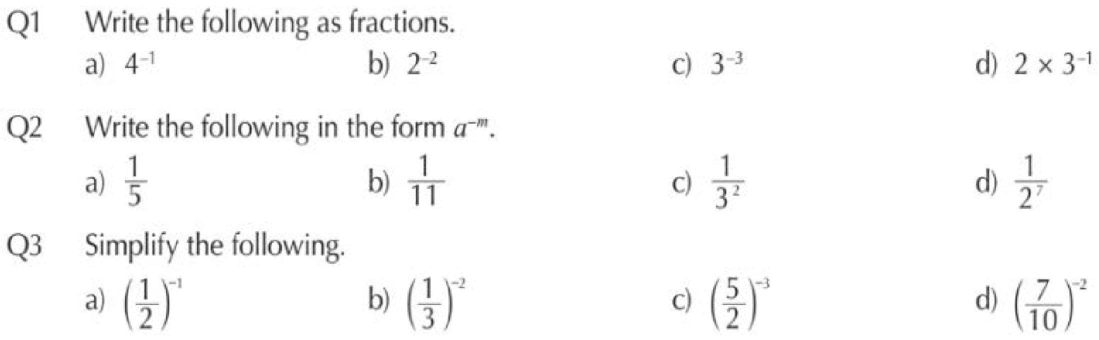
$$5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \mathbf{\frac{1}{125}}$$

Negative Indices

Week 1 Task

HM: 104, 105, 106, 107

Exercise 3



Q1 Write the following as fractions.

a) 4^{-1} b) 2^{-2} c) 3^{-3} d) 2×3^{-1}

Q2 Write the following in the form a^{-m}.

a) $\frac{1}{5}$ b) $\frac{1}{11}$ c) $\frac{1}{3^2}$ d) $\frac{1}{2^7}$

Q3 Simplify the following.

a) $\left(\frac{1}{2}\right)^{-1}$ b) $\left(\frac{1}{3}\right)^{-2}$ c) $\left(\frac{5}{2}\right)^{-3}$ d) $\left(\frac{7}{10}\right)^{-2}$

Negative Indices

Week 1 Task 3

HM: 104, 105, 106, 107

Example 5

Simplify the following: a) $y^4 \div \frac{1}{y^3}$ b) $z^8 \times (z^4)^{-2}$

a)
1. Rewrite $\frac{1}{y^3}$ as a negative index. $y^4 \div \frac{1}{y^3} = y^4 \div y^{-3}$
2. Subtract the indices. $= y^{4-(-3)} = \mathbf{y^7}$

b)
1. Multiply the indices to simplify. $z^8 \times (z^4)^{-2} = z^8 \times z^{4\times(-2)} = z^8 \times z^{-8}$
2. Now add the indices. $= z^{8+(-8)} = z^0$
3. Anything to the power 0 is 1. $= \mathbf{1}$

Negative Indices

Week 1 Task 3

HM: 104, 105, 106, 107

Example 6

Evaluate $2^4 \times 5^{-3}$. Give the answer as a fraction.

1. Turn the negative index into a fraction. $2^4 \times 5^{-3} = 2^4 \times \frac{1}{5^3}$
2. Evaluate the powers. $= \frac{2^4}{5^3} = \mathbf{\frac{16}{125}}$

Negative Indices

Week 1 Task 3

HM: 104, 105, 106, 107

Exercise 4

Q1 Simplify the following. Leave your answers in index form.

a) $5^4 \times 5^{-2}$ b) $g^6 \div g^{-6}$ c) $2^{16} \div \frac{1}{2^4}$ d) $k^{10} \times k^{-6} \div k^0$

e) $\left(\frac{1}{p^4}\right)^5$ f) $\left(\frac{l^{-5}}{l^6}\right)^{-3}$ g) $\frac{n^{-4} \times n}{(n^{-3})^6}$ h) $\left(\frac{10^7 \times 10^{-11}}{10^9 \div 10^4}\right)^{-5}$

Q2 a) Write the number 0.01 as:

(i) a fraction of the form $\frac{1}{a}$ (ii) a fraction of the form $\frac{1}{10^n}$ (iii) a power of 10.

b) Rewrite the following as powers of 10:

(i) 0.1 (ii) 0.00000001 (iii) 0.0001 (iv) 1

Q3 Evaluate the following. Write the answers as fractions.

a) $3^2 \times 5^{-2}$ b) $2^{-3} \times 7^1$ c) $\left(\frac{1}{2}\right)^{-2} \times \left(\frac{1}{3}\right)^2$ d) $6^{-4} \div 6^{-2}$

e) $(-9)^2 \times (-5)^{-3}$ f) $8^{-5} \times 8^3 \times 3^3$ g) $10^{-5} \div 10^6 \times 10^4$ h) $\left(\frac{3}{4}\right)^{-1} \div \left(\frac{1}{2}\right)^{-3}$

Fractional Indices

Week 1 Task 4

HM: 108, 109

If the index is a **fraction**, you can rewrite the power as a **root**. The **denominator** tells you the root to use.

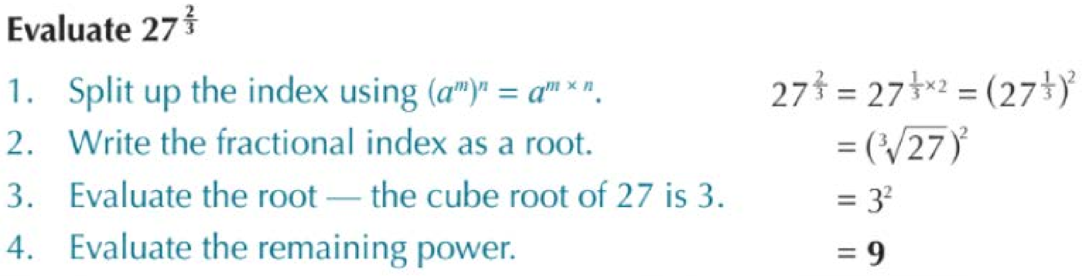
$a^{\frac{1}{2}} = \sqrt{a}$	If the index is $\frac{1}{2}$ then you replace the power with the **square** root $\sqrt{\ }$.
$a^{\frac{1}{3}} = \sqrt[3]{a}$	If the index is $\frac{1}{3}$ then you replace the power with the **cube** root $\sqrt[3]{\ }$.
$a^{\frac{1}{m}} = \sqrt[m]{a}$	More generally, if the index is $\frac{1}{m}$ then you replace the power with the ***m*th root** $\sqrt[m]{\ }$.
$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$	If the numerator **isn't 1**, i.e. the index is $\frac{n}{m}$, you still use the ***m*th root** $\sqrt[m]{\ }$ but you also need to **raise the root** to the power of n.

Fractional Indices

Week 1 Task 4

HM: 108, 109

Example 7



Evaluate $27^{\frac{2}{3}}$

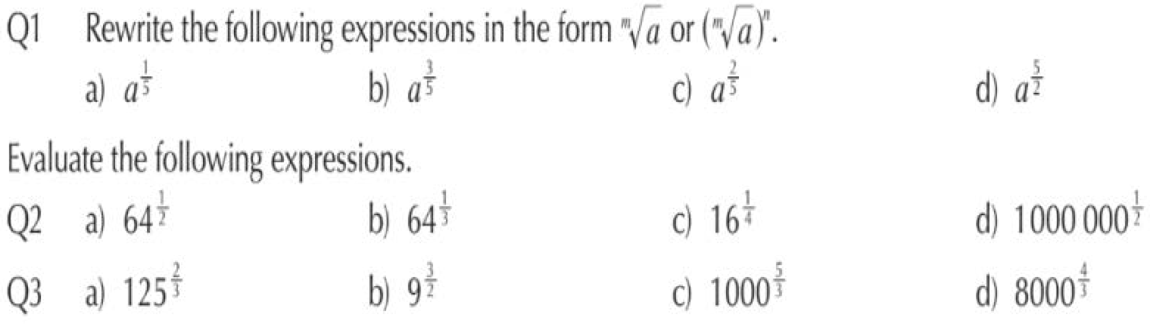
1. Split up the index using $(a^m)^n = a^{m \times n}$. $27^{\frac{2}{3}} = 27^{\frac{1}{3} \times 2} = (27^{\frac{1}{3}})^2$
2. Write the fractional index as a root. $= (\sqrt[3]{27})^2$
3. Evaluate the root — the cube root of 27 is 3. $= 3^2$
4. Evaluate the remaining power. $= \mathbf{9}$

Fractional Indices

Week 1 Task 4

HM: 108, 109

Exercise 5



Q1 Rewrite the following expressions in the form $\sqrt[m]{a}$ or $(\sqrt[m]{a})^n$.

a) $a^{\frac{1}{5}}$ b) $a^{\frac{3}{5}}$ c) $a^{\frac{2}{5}}$ d) $a^{\frac{5}{2}}$

Evaluate the following expressions.

Q2 a) $64^{\frac{1}{2}}$ b) $64^{\frac{1}{3}}$ c) $16^{\frac{1}{4}}$ d) $1000\,000^{\frac{1}{2}}$

Q3 a) $125^{\frac{2}{3}}$ b) $9^{\frac{3}{2}}$ c) $1000^{\frac{5}{3}}$ d) $8000^{\frac{4}{3}}$

Multiplying and Dividing Surds

Week 1 Task 5

HM: 113, 114

To **multiply** or **divide** two surds, combine them into a **single surd** using the rules below.

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ e.g. $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$. Also, $(\sqrt{b})^2 = \sqrt{b} \times \sqrt{b} = \sqrt{b \times b} = \sqrt{b^2} = b$

$\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ e.g. $\sqrt{8} \div \sqrt{2} = \frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$

This is useful for **simplifying** expressions containing surds — the aim is to make the number under the root as **small** as possible or **get rid** of the root completely. To do this, split the number up into two **factors**, one of which should be a **square number**. You can then take the **square root** of this square number to simplify. When you're done, you'll be left with an expression of the form $a\sqrt{b}$ where a and b are **integers** and b is as small as possible.

Multiplying and Dividing Surds

Week 1 Task 5

HM: 113, 114

Example 1

Simplify $\sqrt{72}$.

1. Break 72 down into factors — one of them needs to be a square number. $\sqrt{72} = \sqrt{36 \times 2}$
2. Write as two roots multiplied together. $= \sqrt{36} \times \sqrt{2}$
3. Evaluate $\sqrt{36}$. $= \mathbf{6\sqrt{2}}$

Multiplying and Dividing Surds

Week 1 Task 5

HM: 113, 114

Example 2

Find $\sqrt{5} \times \sqrt{15}$. Simplify your answer.

1. Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$.
2. Now find factors of 75 so you can simplify — remember that one of the factors needs to be a square number.

$$\sqrt{5} \times \sqrt{15} = \sqrt{5 \times 15} = \sqrt{75}$$
$$= \sqrt{25 \times 3}$$
$$= \sqrt{25} \times \sqrt{3}$$
$$= \mathbf{5\sqrt{3}}$$

Multiplying and Dividing Surds

Week 1 Task 5

HM: 113, 114

Exercise 1

Q1 Simplify:

a) $\sqrt{12}$ b) $\sqrt{20}$ c) $\sqrt{50}$ d) $\sqrt{32}$

e) $\sqrt{108}$ f) $\sqrt{300}$ g) $\sqrt{98}$ h) $\sqrt{192}$

Q2 Rewrite the following in the form $a\sqrt{b}$, where a and b are integers. Simplify your answers where possible.

a) $\sqrt{2} \times \sqrt{24}$ b) $\sqrt{3} \times \sqrt{12}$ c) $\sqrt{3} \times \sqrt{24}$

d) $\sqrt{2} \times \sqrt{10}$ e) $\sqrt{40} \times \sqrt{2}$ f) $\sqrt{3} \times \sqrt{60}$

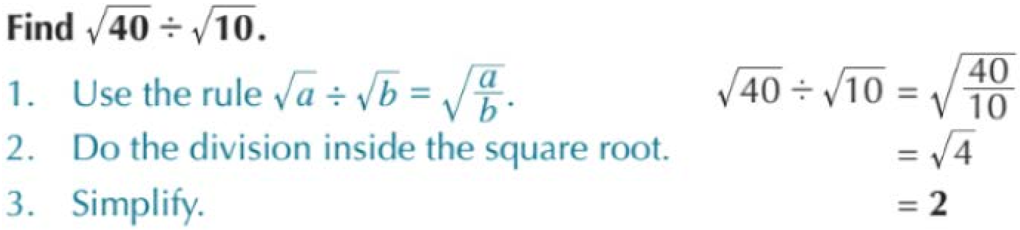
g) $\sqrt{7} \times \sqrt{35}$ h) $\sqrt{50} \times \sqrt{10}$ i) $\sqrt{8} \times \sqrt{24}$

Multiplying and Dividing Surds

Week 1 Task 5

HM: 113, 114

Example 3



Find $\sqrt{40} \div \sqrt{10}$.

1. Use the rule $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.
2. Do the division inside the square root.
3. Simplify.

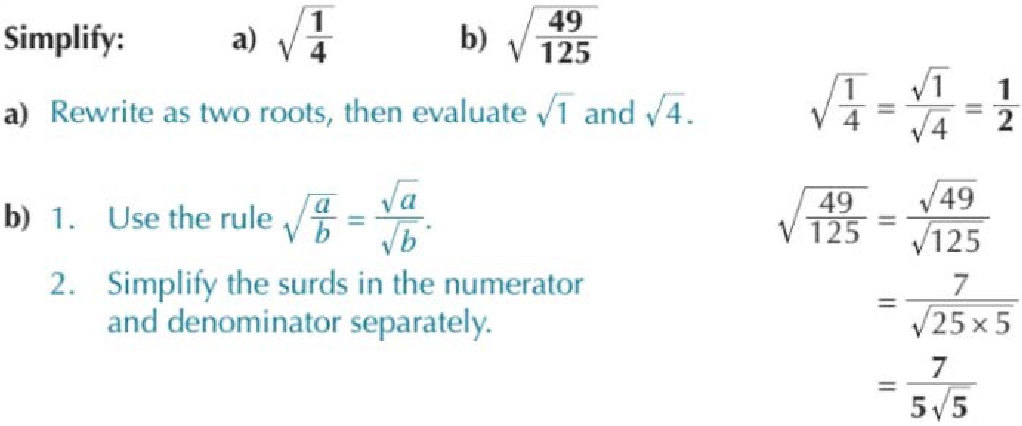
$$\sqrt{40} \div \sqrt{10} = \sqrt{\frac{40}{10}}$$
$$= \sqrt{4}$$
$$= \mathbf{2}$$

Multiplying and Dividing Surds

Week 1 Task 5

HM: 113, 114

Example 4



Simplify: **a)** $\sqrt{\frac{1}{4}}$ **b)** $\sqrt{\frac{49}{125}}$

a) Rewrite as two roots, then evaluate $\sqrt{1}$ and $\sqrt{4}$.

$$\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{\mathbf{1}}{\mathbf{2}}$$

b) 1. Use the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

2. Simplify the surds in the numerator and denominator separately.

$$\sqrt{\frac{49}{125}} = \frac{\sqrt{49}}{\sqrt{125}}$$
$$= \frac{7}{\sqrt{25 \times 5}}$$
$$= \frac{\mathbf{7}}{\mathbf{5\sqrt{5}}}$$

Multiplying and Dividing Surds

Week 1 Task 5

HM: 113, 114

Exercise 2

Q1 Calculate the exact values of the following. Simplify your answers where possible.

a) $\sqrt{90} \div \sqrt{10}$ b) $\sqrt{72} \div \sqrt{2}$ c) $\sqrt{200} \div \sqrt{8}$ d) $\sqrt{243} \div \sqrt{3}$

e) $\sqrt{294} \div \sqrt{6}$ f) $\sqrt{80} \div \sqrt{10}$ g) $\sqrt{120} \div \sqrt{10}$ h) $\sqrt{180} \div \sqrt{3}$

i) $\sqrt{180} \div \sqrt{9}$ j) $\sqrt{96} \div \sqrt{6}$ k) $\sqrt{484} \div \sqrt{22}$ l) $\sqrt{210} \div \sqrt{35}$

Q2 Simplify the following as far as possible.

a) $\sqrt{\frac{1}{9}}$ b) $\sqrt{\frac{4}{25}}$ c) $\sqrt{\frac{49}{121}}$

d) $\sqrt{\frac{100}{64}}$ e) $\sqrt{\frac{18}{200}}$ f) $\sqrt{\frac{2}{25}}$

g) $\sqrt{\frac{108}{147}}$ h) $\sqrt{\frac{27}{64}}$ i) $\sqrt{\frac{98}{121}}$

Adding and Subtracting Surds

Week 1 Task 6

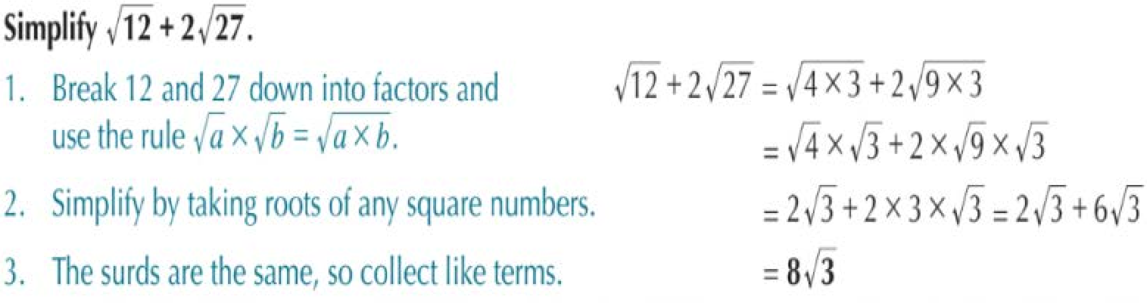
HM: 115

You can simplify expressions containing surds by **collecting like terms** — but you can **only** add or subtract terms where the number under the root is **the same**. So you can do $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$ but $\sqrt{2} + \sqrt{3}$ can't be simplified — $\sqrt{a} + \sqrt{b}$ **doesn't equal** $\sqrt{a+b}$. You'll probably have to **simplify** individual terms first to make the surd parts match.

Adding and Subtracting Surds

Week 1 Task 6
HM: 115

Example 5



Simplify $\sqrt{12} + 2\sqrt{27}$.

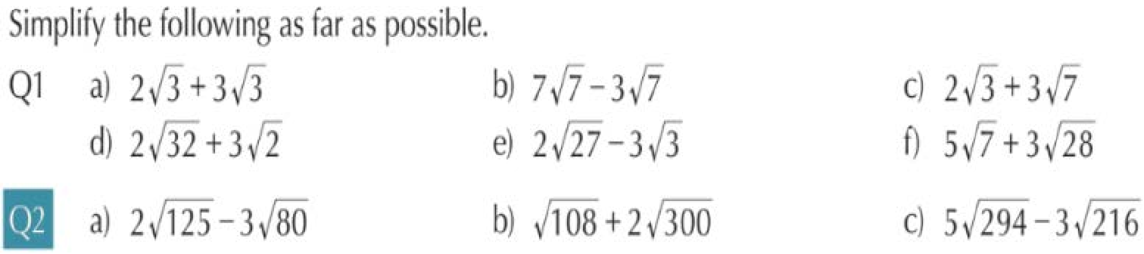
1. Break 12 and 27 down into factors and use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$.
2. Simplify by taking roots of any square numbers.
3. The surds are the same, so collect like terms.

$$\sqrt{12} + 2\sqrt{27} = \sqrt{4 \times 3} + 2\sqrt{9 \times 3}$$
$$= \sqrt{4} \times \sqrt{3} + 2 \times \sqrt{9} \times \sqrt{3}$$
$$= 2\sqrt{3} + 2 \times 3 \times \sqrt{3} = 2\sqrt{3} + 6\sqrt{3}$$
$$= \mathbf{8\sqrt{3}}$$

Adding and Subtracting Surds

Week 1 Task 6
HM: 115

Exercise 3

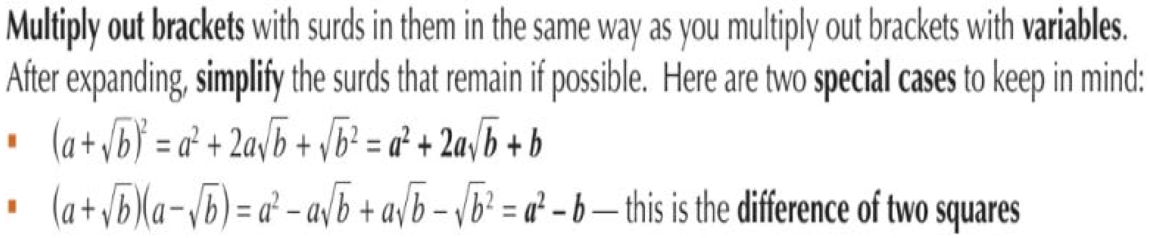


Simplify the following as far as possible.

Q1 a) $2\sqrt{3} + 3\sqrt{3}$ b) $7\sqrt{7} - 3\sqrt{7}$ c) $2\sqrt{3} + 3\sqrt{7}$
d) $2\sqrt{32} + 3\sqrt{2}$ e) $2\sqrt{27} - 3\sqrt{3}$ f) $5\sqrt{7} + 3\sqrt{28}$

Q2 a) $2\sqrt{125} - 3\sqrt{80}$ b) $\sqrt{108} + 2\sqrt{300}$ c) $5\sqrt{294} - 3\sqrt{216}$

Multiplying Brackets using Surds

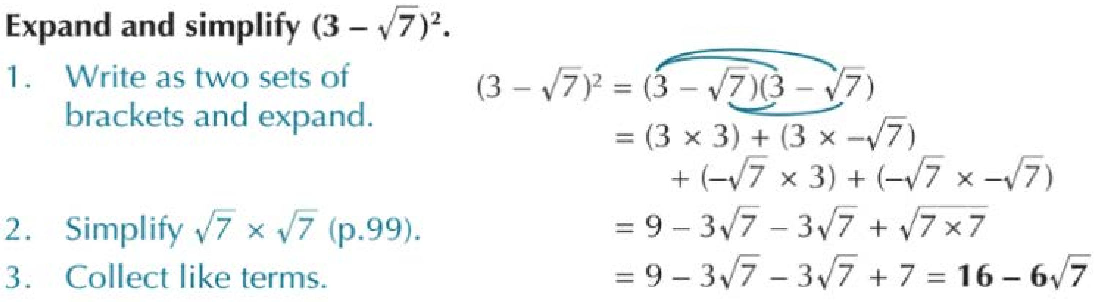


Multiply out brackets with surds in them in the same way as you multiply out brackets with **variables**. After expanding, **simplify** the surds that remain if possible. Here are two **special cases** to keep in mind:

- $(a+\sqrt{b})^2 = a^2 + 2a\sqrt{b} + \sqrt{b}^2 = \mathbf{a^2 + 2a\sqrt{b} + b}$
- $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - a\sqrt{b} + a\sqrt{b} - \sqrt{b}^2 = \mathbf{a^2 - b}$ — this is the **difference of two squares**

Multiplying Brackets using Surds

Example 6



Expand and simplify $(3 - \sqrt{7})^2$.

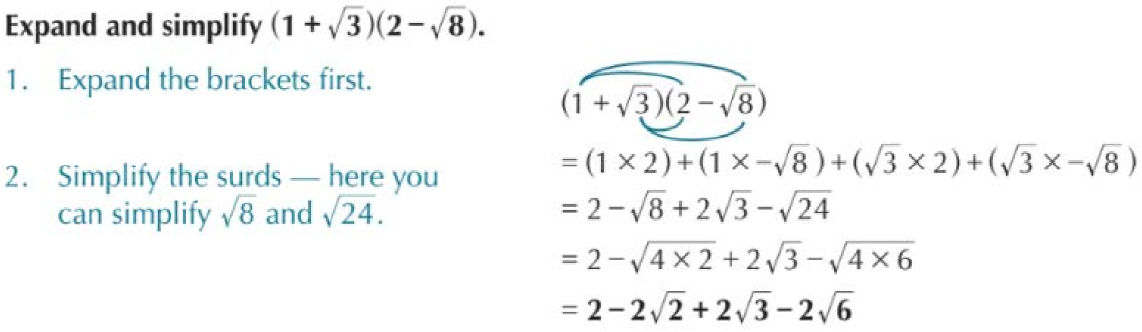
1. Write as two sets of brackets and expand.
2. Simplify $\sqrt{7} \times \sqrt{7}$ (p.99).
3. Collect like terms.

$$(3 - \sqrt{7})^2 = (3 - \sqrt{7})(3 - \sqrt{7})$$
$$= (3 \times 3) + (3 \times -\sqrt{7}) + (-\sqrt{7} \times 3) + (-\sqrt{7} \times -\sqrt{7})$$
$$= 9 - 3\sqrt{7} - 3\sqrt{7} + \sqrt{7 \times 7}$$
$$= 9 - 3\sqrt{7} - 3\sqrt{7} + 7 = \mathbf{16 - 6\sqrt{7}}$$

Multiplying Brackets using Surds

Week 1 Task 7
HM: 116, 117

Example 7



Expand and simplify $(1+\sqrt{3})(2-\sqrt{8})$.

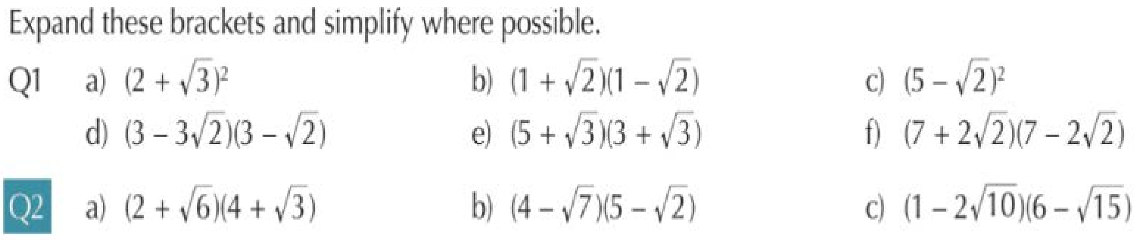
1. Expand the brackets first.
2. Simplify the surds — here you can simplify $\sqrt{8}$ and $\sqrt{24}$.

$$(1+\sqrt{3})(2-\sqrt{8})$$
$$=(1\times 2)+(1\times -\sqrt{8})+(\sqrt{3}\times 2)+(\sqrt{3}\times -\sqrt{8})$$
$$=2-\sqrt{8}+2\sqrt{3}-\sqrt{24}$$
$$=2-\sqrt{4\times 2}+2\sqrt{3}-\sqrt{4\times 6}$$
$$=\mathbf{2-2\sqrt{2}+2\sqrt{3}-2\sqrt{6}}$$

Multiplying Brackets using Surds

Week 1 Task 7
HM: 116, 117

Exercise 4



Expand these brackets and simplify where possible.

Q1 a) $(2+\sqrt{3})^2$ b) $(1+\sqrt{2})(1-\sqrt{2})$ c) $(5-\sqrt{2})^2$
d) $(3-3\sqrt{2})(3-\sqrt{2})$ e) $(5+\sqrt{3})(3+\sqrt{3})$ f) $(7+2\sqrt{2})(7-2\sqrt{2})$

Q2 a) $(2+\sqrt{6})(4+\sqrt{3})$ b) $(4-\sqrt{7})(5-\sqrt{2})$ c) $(1-2\sqrt{10})(6-\sqrt{15})$

Rationalising the Denominator

Week 1 Task 8
HM: 118

'**Rationalising the denominator**' means 'getting rid of surds from the bottom of a fraction'. For the simplest type, you do this by **multiplying** the top and bottom of the fraction by the surd.

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{\sqrt{b} \times \sqrt{b}} = \frac{a\sqrt{b}}{b}$$

Rationalising the Denominator

Week 1 Task 8
HM: 118

Example 8

Rationalise the denominators of: a) $\frac{5}{2\sqrt{15}}$ b) $\frac{2}{\sqrt{8}}$

a) 1. Multiply by $\frac{\sqrt{15}}{\sqrt{15}}$ to eliminate $\sqrt{15}$ from the denominator. Remember that $\sqrt{15} \times \sqrt{15} = 15$.

$$\frac{5}{2\sqrt{15}} = \frac{5}{2\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}}$$

$$= \frac{5\sqrt{15}}{2\sqrt{15} \times \sqrt{15}} = \frac{5\sqrt{15}}{2 \times 15}$$

2. Simplify the fraction.

$$= \frac{5\sqrt{15}}{30} = \mathbf{\frac{\sqrt{15}}{6}}$$

b) 1. Multiply by $\frac{\sqrt{8}}{\sqrt{8}}$.

$$\frac{2}{\sqrt{8}} = \frac{2}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{2\sqrt{8}}{\sqrt{8} \times \sqrt{8}}$$

2. Simplify the surd.

$$= \frac{2\sqrt{4 \times 2}}{8} = \frac{2 \times 2\sqrt{2}}{8}$$

3. Simplify the fraction.

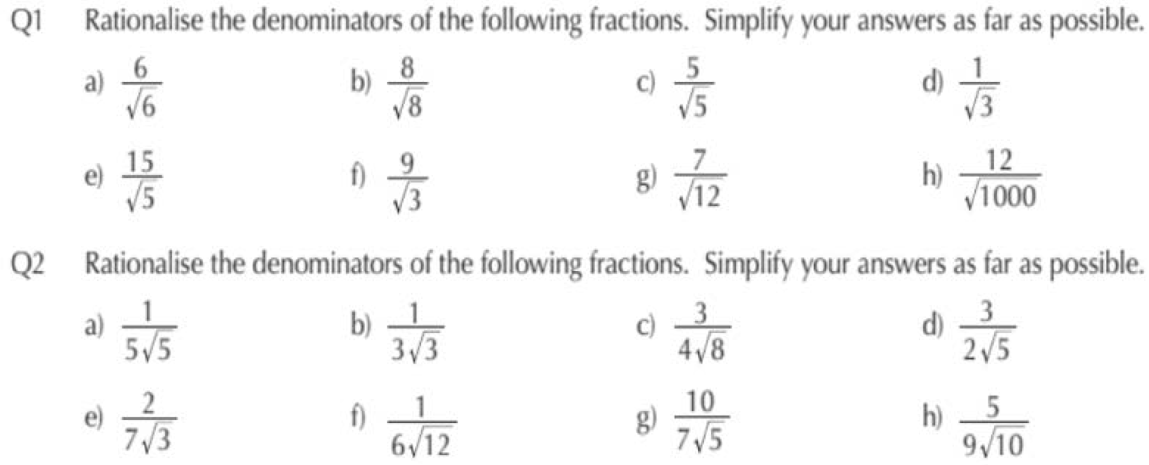
$$= \frac{4\sqrt{2}}{8} = \mathbf{\frac{\sqrt{2}}{2}}$$

Rationalising the Denominator

Week 1 Task 8

HM: 118

Exercise 5



Q1 Rationalise the denominators of the following fractions. Simplify your answers as far as possible.

a) $\frac{6}{\sqrt{6}}$ b) $\frac{8}{\sqrt{8}}$ c) $\frac{5}{\sqrt{5}}$ d) $\frac{1}{\sqrt{3}}$

e) $\frac{15}{\sqrt{5}}$ f) $\frac{9}{\sqrt{3}}$ g) $\frac{7}{\sqrt{12}}$ h) $\frac{12}{\sqrt{1000}}$

Q2 Rationalise the denominators of the following fractions. Simplify your answers as far as possible.

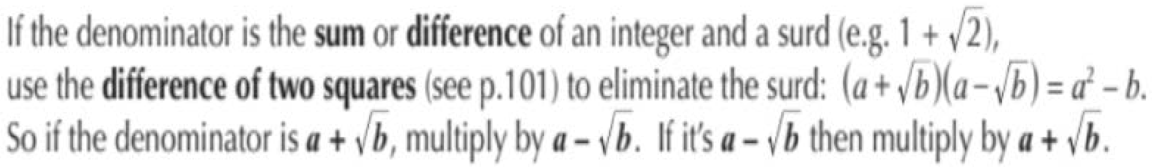
a) $\frac{1}{5\sqrt{5}}$ b) $\frac{1}{3\sqrt{3}}$ c) $\frac{3}{4\sqrt{8}}$ d) $\frac{3}{2\sqrt{5}}$

e) $\frac{2}{7\sqrt{3}}$ f) $\frac{1}{6\sqrt{12}}$ g) $\frac{10}{7\sqrt{5}}$ h) $\frac{5}{9\sqrt{10}}$

Rationalising the Denominator

Week 1 Task 8

HM: 118



If the denominator is the **sum** or **difference** of an integer and a surd (e.g. $1 + \sqrt{2}$),
use the **difference of two squares** (see p.101) to eliminate the surd: $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$.
So if the denominator is $\boldsymbol{a + \sqrt{b}}$, multiply by $\boldsymbol{a - \sqrt{b}}$. If it's $\boldsymbol{a - \sqrt{b}}$ then multiply by $\boldsymbol{a + \sqrt{b}}$.

Rationalising the Denominator

Week 1 Task 8

HM: 118

Example 9

Rationalise the denominator of $\frac{2+2\sqrt{2}}{1-\sqrt{2}}$.

1. Multiply top and bottom by $(1+\sqrt{2})$ to get rid of the surd in the denominator.

$$\frac{2+2\sqrt{2}}{1-\sqrt{2}} = \frac{(2+2\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$$

2. Expand the brackets in the numerator and denominator

$$= \frac{2+2\sqrt{2}+2\sqrt{2}+4}{1+\sqrt{2}-\sqrt{2}-2}$$

3. Simplify any remaining surds and the fraction.

$$= \frac{6+4\sqrt{2}}{-1} = \mathbf{-6-4\sqrt{2}}$$

Rationalising the Denominator

Week 1 Task 8

HM: 118

Exercise 6

Q1 Rationalise the denominators of the following fractions. Simplify your answers as far as possible.

a) $\frac{1}{2+\sqrt{2}}$ b) $\frac{5}{1-\sqrt{7}}$ c) $\frac{10}{5+\sqrt{11}}$ d) $\frac{9}{12-3\sqrt{17}}$

Q2 Rewrite the following as fractions with rational denominators in their simplest form.

a) $\frac{\sqrt{2}}{2+3\sqrt{2}}$ b) $\frac{1+\sqrt{2}}{1-\sqrt{2}}$ c) $\frac{2+\sqrt{3}}{1-\sqrt{3}}$

d) $\frac{1-\sqrt{5}}{2-\sqrt{5}}$ e) $\frac{1+2\sqrt{2}}{1-2\sqrt{2}}$ f) $\frac{7+8\sqrt{2}}{9+5\sqrt{2}}$

Q3 Show that $\frac{1}{1-\frac{1}{\sqrt{2}}}$ can be written as $2+\sqrt{2}$.

Q4 Show that $\frac{1}{1+\frac{1}{\sqrt{3}}}$ can be written as $\frac{3-\sqrt{3}}{2}$.

Week 1 Assessment

Question 1

Simplify:

a) $2x^3 \times 4x^4$

[1 mark]

b) $(3y^2)^4$

[2 marks]

c) $5z^0$

[1 mark]

Week 1 Assessment

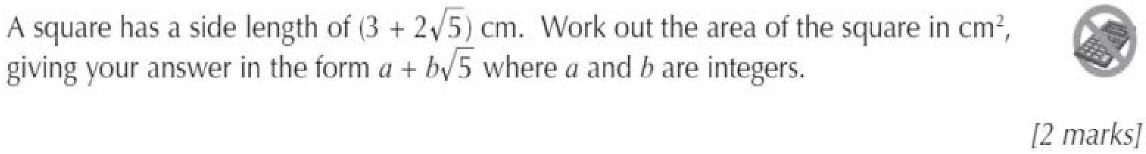
Question 2

Showing every step of your working, prove that $\left(\frac{4}{9}\right)^{-\frac{3}{2}} = 3\frac{3}{8}$.

[3 marks]

Week 1 Assessment

Question 3

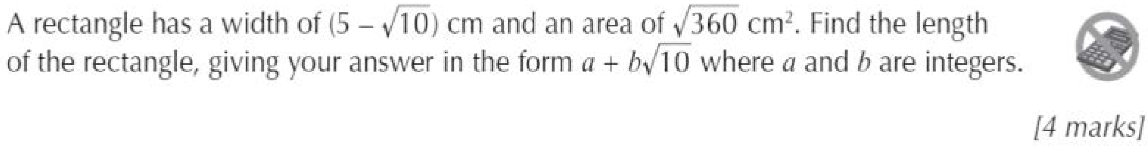


A square has a side length of $(3 + 2\sqrt{5})$ cm. Work out the area of the square in cm^2, giving your answer in the form $a + b\sqrt{5}$ where a and b are integers.

[2 marks]

Week 1 Assessment

Question 4



A rectangle has a width of $(5 - \sqrt{10})$ cm and an area of $\sqrt{360}$ cm^2. Find the length of the rectangle, giving your answer in the form $a + b\sqrt{10}$ where a and b are integers.

[4 marks]

Week 1 Assessment

Question 5

$x = y \times 10^z$ where $4 < y < 10$

Find an expression for x^2 in standard form. PROBLEM SOLVING

[3 marks]

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Year 11 Transition Tasks to A Level Mathematics

Week 2 Tasks

1. Rearranging Formulae HM: 280, 281, 282, 283, 284, 285, 286

2. Identities HM: 154

3. Solving Quadratic Equation by Factorising HM: 230, 231, 232, 233, 234

4. Completing the Square HM: 235, 236, 237, 238, 239

5. The Quadratic Formula HM: 240, 241, 242

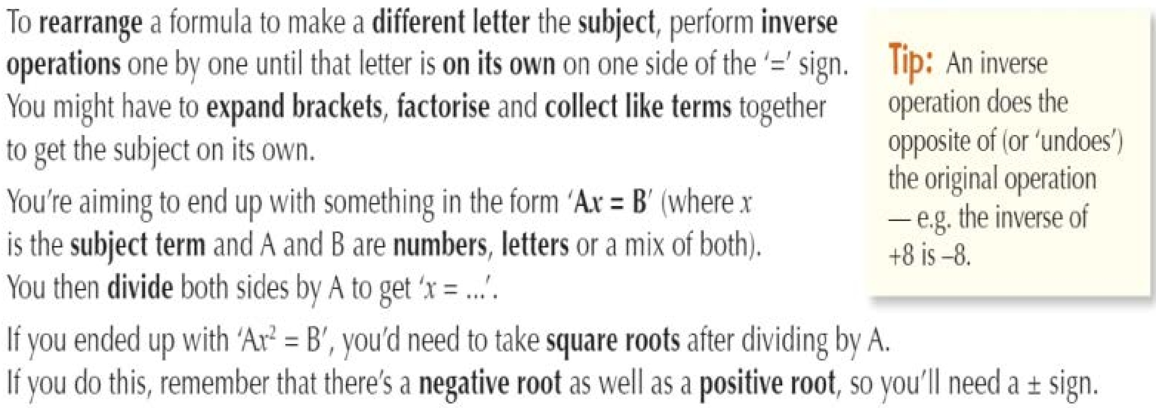
6. Simultaneous Linear Equations HM: 191, 192, 193, 194

7. Simultaneous Linear and Quadratic Equations HM: 246

Rearranging Formulae

Week 2 Task 1

HM: 280, 281, 282, 283, 284, 285, 286



To **rearrange** a formula to make a **different letter** the **subject**, perform **inverse operations** one by one until that letter is **on its own** on one side of the '=' sign. You might have to **expand brackets**, **factorise** and **collect like terms** together to get the subject on its own.

You're aiming to end up with something in the form '**Ax = B**' (where x is the **subject term** and A and B are **numbers**, **letters** or a mix of both). You then **divide** both sides by A to get 'x = ...'.

Tip: An inverse operation does the opposite of (or 'undoes') the original operation — e.g. the inverse of +8 is –8.

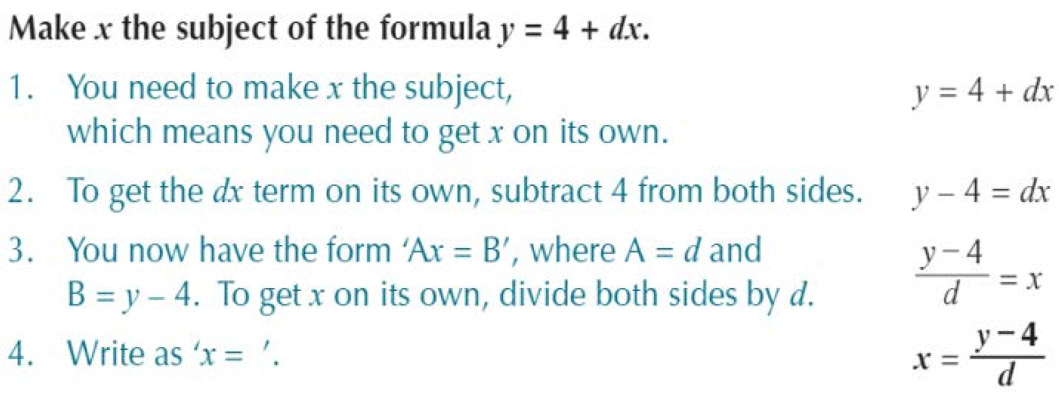
If you ended up with 'Ax^2 = B', you'd need to take **square roots** after dividing by A. If you do this, remember that there's a **negative root** as well as a **positive root**, so you'll need a ± sign.

Rearranging Formulae

Week 2 Task 1

HM: 280, 281, 282, 283, 284, 285, 286

Example 1



Make x the subject of the formula $y = 4 + dx$.

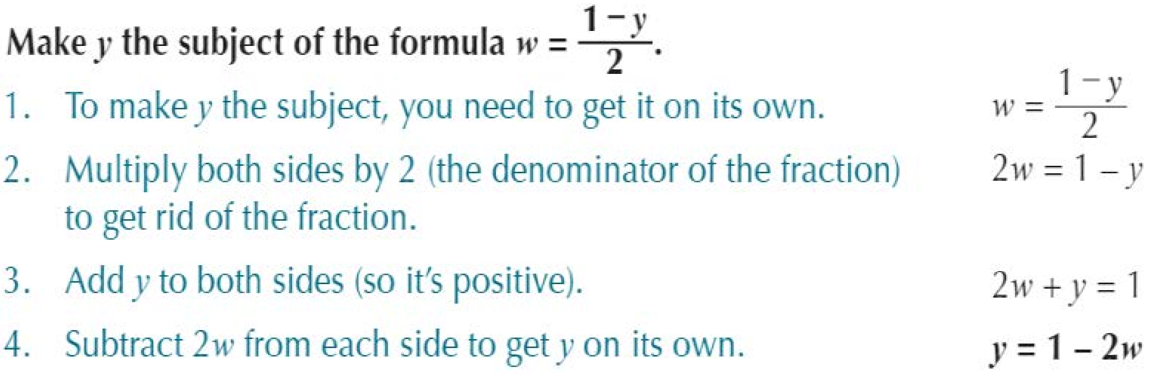
1. You need to make x the subject, which means you need to get x on its own. $y = 4 + dx$
2. To get the dx term on its own, subtract 4 from both sides. $y - 4 = dx$
3. You now have the form 'Ax = B', where A = d and B = y – 4. To get x on its own, divide both sides by d. $\frac{y-4}{d} = x$
4. Write as 'x = '. $x = \frac{y-4}{d}$

Rearranging Formulae

Week 2 Task 1

HM: 280, 281, 282, 283, 284, 285, 286

Example 2



Make y the subject of the formula $w = \frac{1-y}{2}$.

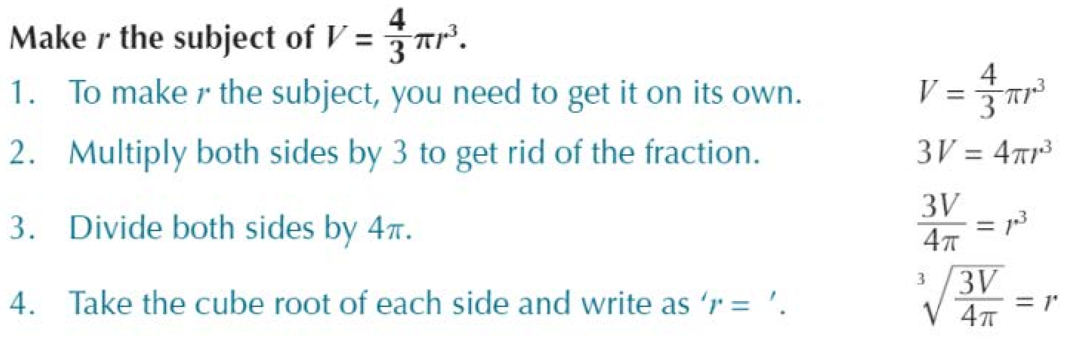
1. To make y the subject, you need to get it on its own. $w = \frac{1-y}{2}$
2. Multiply both sides by 2 (the denominator of the fraction) to get rid of the fraction. $2w = 1 - y$
3. Add y to both sides (so it's positive). $2w + y = 1$
4. Subtract $2w$ from each side to get y on its own. $\mathbf{y = 1 - 2w}$

Rearranging Formulae

Week 2 Task 1

HM: 280, 281, 282, 283, 284, 285, 286

Example 3



Make r the subject of $V = \frac{4}{3}\pi r^3$.

1. To make r the subject, you need to get it on its own. $V = \frac{4}{3}\pi r^3$
2. Multiply both sides by 3 to get rid of the fraction. $3V = 4\pi r^3$
3. Divide both sides by 4π. $\frac{3V}{4\pi} = r^3$
4. Take the cube root of each side and write as '$r =$ '. $\sqrt[3]{\frac{3V}{4\pi}} = r$

$$\Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$$

Rearranging Formulae

Example 4

If the subject appears **more than once**, you're going to have to do some **factorising**

Make a the subject of the formula $x(a + 1) = 3(1 - 2a)$.

1. Multiply out the brackets. $x(a + 1) = 3(1 - 2a)$

   $ax + x = 3 - 6a$
2. Collect all the a terms on one side and the non-a terms on the other. $ax + 6a = 3 - x$
3. Factorise the left-hand side to get it into the form to 'Aa = B'. $a(x + 6) = 3 - x$
4. Divide by $(x + 6)$ to get a on its own. $\boldsymbol{a = \frac{3 - x}{x + 6}}$

Rearranging Formulae

Exercise 1

Q1 Make x the subject of each of the following formulas.

a) $y = x + 2$
b) $2z = 3r + x$
c) $y = 4x$
d) $k = 2(1 + 2x)$
e) $v = \frac{2}{3}x - 2$
f) $y + 1 = \frac{x - 1}{3}$

Q2 Consider the formula $w = \frac{1}{1 + y}$.

a) Multiply both sides of the formula by $1 + y$.
b) Hence make y the subject of the formula.

Q3 Make y the subject of the following formulas.

a) $w = \frac{3}{2y}$
b) $z + 2 = \frac{2}{1 - y}$
c) $uv = \frac{1}{1 - 2y}$
d) $a + b = \frac{2}{4 - 3y}$

Q4 Consider the formula $2k = 12 - \sqrt{w - 2}$.

a) Make $\sqrt{w - 2}$ the subject of the formula.
b) By first squaring both sides of your answer to part a), make w the subject of the formula.

Rearranging Formulae

Week 2 Task 1

HM: 280, 281, 282, 283, 284, 285, 286

Exercise 1

Q5 Make w the subject of the following formulas.

a) $a = \sqrt{w}$ b) $x = 1 + \sqrt{w}$ c) $y = \sqrt{w-2}$

d) $f - 3 = 2\sqrt{w}$ e) $j = \sqrt{3+4w}$ f) $a = \sqrt{1-2w}$

Q6 Consider the formula $t = 1 - 3(z + 1)^2$.

a) Make $(z + 1)^2$ the subject of the formula.

b) By first square rooting both sides of your answer to part a), make z the subject of the formula.

Q7 Make z the subject of the following formulas.

a) $x = 1 + z^2$ b) $2t = 3 - z^2$ c) $xy = 1 - 4z^2$

d) $t + 2 = 3(z - 2)^2$ e) $g = 4 - (2z + 3)^2$ f) $r = 4 - 2(5 - 3z)^2$

Q8 Make a the subject of the following formulas.

a) $x(a + b) = a - 1$ b) $x - ab = c - ad$ c) $c = \dfrac{1+a}{1-2a}$ d) $2e = \dfrac{2+3a}{a}$

Identities

Week 2 Task 2

HM: 154

An **equation** is a way of showing that two expressions are equal for some particular values of an unknown.
Identities are like equations, but are **always true**, for **any value** of the unknown.
Identities have the symbol '≡' instead of '='.

E.g. $x - 1 = 2$ is an equation — it's only true when $x = 3$.
$x + 1 \equiv 1 + x$ is an identity — it's always true, whatever the value of x.

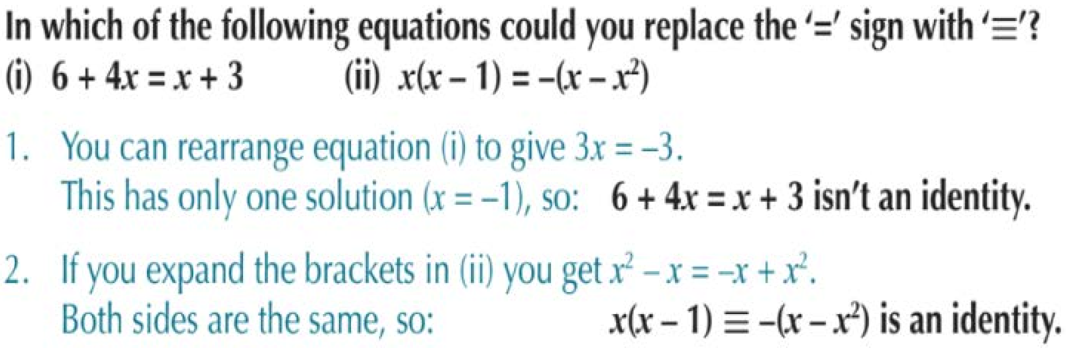
In identity questions, you should **rearrange** the expressions on **either side** to see if they're the **same**.
You **don't** need to take things to the other side, like you would if you were solving an equation.

Identities

Week 2 Task 2

HM: 154

Example 1



In which of the following equations could you replace the '=' sign with '≡'?

(i) $6 + 4x = x + 3$ (ii) $x(x - 1) = -(x - x^2)$

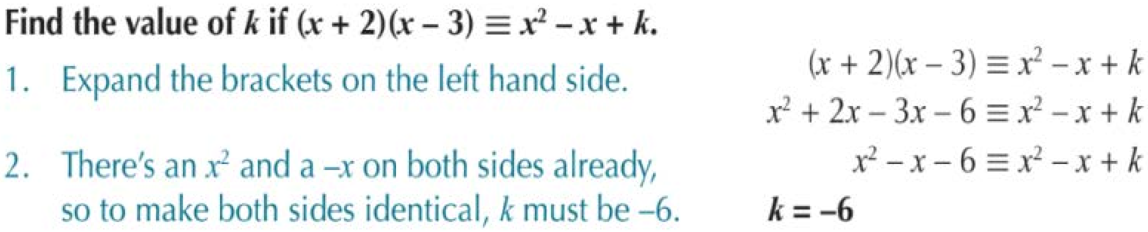
1. You can rearrange equation (i) to give $3x = -3$.
   This has only one solution ($x = -1$), so: **$6 + 4x = x + 3$ isn't an identity.**
2. If you expand the brackets in (ii) you get $x^2 - x = -x + x^2$.
   Both sides are the same, so: **$x(x - 1) \equiv -(x - x^2)$ is an identity.**

Identities

Week 2 Task 2

HM: 154

Example 2



Find the value of k if $(x + 2)(x - 3) \equiv x^2 - x + k$.

1. Expand the brackets on the left hand side.
2. There's an x^2 and a $-x$ on both sides already, so to make both sides identical, k must be -6.

$$(x + 2)(x - 3) \equiv x^2 - x + k$$
$$x^2 + 2x - 3x - 6 \equiv x^2 - x + k$$
$$x^2 - x - 6 \equiv x^2 - x + k$$

$k = -6$

Rearranging Formulae

Week 2 Task 2

HM: 154

Exercise 1

Q1 For each of the following, state whether or not you could you replace the box with the symbol '≡'.

a) $x - 1 \square 0$
b) $x^2 - 3 \square 3 - x^2$
c) $3(x + 2) - x \square 2(x + 3)$
d) $x^2 + 2x + 1 \square (x + 1)^2$
e) $4(2 - x) \square 2(4 - 2x)$
f) $4x^2 - x \square 2(x^2 - 2x)$

Q2 Find the value of a if:

a) $2(x + 5) \equiv 2x + 1 + a$
b) $ax + 3 \equiv 5x + 2 - (x - 1)$
c) $(x + 4)(x - 1) \equiv x^2 + ax - 4$
d) $(x + 2)^2 \equiv x^2 + 4x + a$
e) $4 - x^2 \equiv (a + x)(a - x)$
f) $(2x - 1)(3 - x) \equiv ax^2 + 7x - 3$

Q3 Prove that: a) $(x + 5)^2 + 3(x - 1)^2 \equiv 4(x^2 + x + 7)$ b) $3(x + 2)^2 - (x - 4)^2 \equiv 2(x^2 + 10x - 2)$

Quadratic Equations by Factorising

Week 2 Task 3

HM: 230, 231, 232, 233, 234

If you know that the **product** of two numbers is equal to 0 (i.e. $p \times q = 0$) then one of the **factors** must be equal to 0 — either $p = 0$ or $q = 0$. You can use this principle to help you to solve a quadratic equation.

Start by **factorising** the quadratic (p.82-84) so that it's in the form $\boldsymbol{(x \pm m)(x \pm n) = 0}$. Then one of the factors must be 0 — either $\boldsymbol{x \pm m = 0}$ or $\boldsymbol{x \pm n = 0}$. Solve each of these for x to give $\boldsymbol{x = \pm m}$ or $\boldsymbol{x = \pm n}$.

If the coefficient of x^2 isn't **1** then the factorisation will be of the form $\boldsymbol{(px \pm q)(rx \pm s) = 0}$. The method to find x is still **the same** though — set **each factor** equal to 0 ($px \pm q = 0$ or $rx \pm s = 0$) and **solve for** $\boldsymbol{x}$.

There are a couple of **special cases**. Firstly, if a quadratic factorises to the form $(px \pm q)^2$, then there's only **one solution** (found by solving $px \pm q = 0$). Secondly, if a quadratic factorises to the form $rx(px \pm q)$, you'll get two solutions — one from solving $px \pm q = 0$ and one from solving $rx = 0$, i.e. $\boldsymbol{x = 0}$.

Quadratic Equations by Factorising

Week 2 Task 3

HM: 230, 231, 232, 233, 234

Example 1

a) Solve the equation $x^2 - 3x + 2 = 0$.

1. Factorise the left-hand side.	$(x-1)(x-2)=0$
2. Set each factor equal to 0.	$x-1=0$ or $x-2=0$
3. Solve to find the two possible values of x.	$\mathbf{x=1}$ or $\mathbf{x=2}$

b) Solve the equation $8x^2 + 6x - 9 = 0$.

1. Factorise the left-hand side.	$(2x+3)(4x-3)=0$
2. Set each factor equal to 0.	$2x+3=0$ or $4x-3=0$
3. Solve to find the two possible values of x.	$2x=-3$ or $4x=3$ $\mathbf{x=-\frac{3}{2}}$ or $\mathbf{x=\frac{3}{4}}$

Quadratic Equations by Factorising

Week 2 Task 3

HM: 230, 231, 232, 233, 234

Exercise 1

Q1 Find the possible values of x for each of the following.

a) $x(x+8)=0$
b) $(x-5)(x-1)=0$
c) $(x+2)(x+6)=0$
d) $(x-9)(x+7)=0$

Q2 Solve the following quadratic equations by factorising.

a) $x^2-3x=0$
b) $x^2+12x=0$
c) $x^2+3x+2=0$
d) $x^2-2x+1=0$
e) $x^2+4x+4=0$
f) $x^2+3x-4=0$
g) $x^2-3x-4=0$
h) $x^2+5x+4=0$
i) $x^2-5x+6=0$
j) $x^2+8x+12=0$
k) $x^2-2x-24=0$
l) $x^2-15x+36=0$

Q3 Find the possible values of x for each of the following.

a) $x(2x-3)=0$
b) $(x-2)(3x-1)=0$
c) $(3x+4)(2x+5)=0$
d) $(4x-7)(5x+2)=0$

Q4 Solve the following equations by factorising.

a) $3x^2+5x=0$
b) $2x^2+x-3=0$
c) $5x^2+3x-2=0$
d) $3x^2-11x+6=0$
e) $4x^2+17x+4=0$
f) $6x^2+x-22=0$
g) $4x^2-20x+25=0$
h) $9x^2-12x+4=0$

Quadratic Equations by Factorising

Week 2 Task 3

HM: 230, 231, 232, 233, 234

It's important that one side of the equation is **0** so it's in the form $ax^2 + bx + c = 0$ **before** you factorise (b or c might be zero). If not, you'll have to **rearrange** into this form by expanding any brackets, getting rid of any fractions, etc. Only then should you factorise your quadratic.

Quadratic Equations by Factorising

Week 2 Task 3

HM: 230, 231, 232, 233, 234

Example 2

a) Solve the equation $12x^2 - 8x = 15$.

1. Rearrange the equation so it's in the form $ax^2 + bx + c = 0$.	$12x^2 - 8x - 15 = 0$
2. Factorise the left-hand side of the equation.	$(2x - 3)(6x + 5) = 0$
3. Set each factor equal to 0.	$2x - 3 = 0$ or $6x + 5 = 0$
4. Solve for each possible value of x.	$\mathbf{x = \frac{3}{2}}$ or $\mathbf{x = -\frac{5}{6}}$

b) Solve the equation $x = \dfrac{-(x+1)}{x-3}$.

1. Get rid of the fraction by multiplying both sides by $x - 3$.	$x(x - 3) = -(x + 1)$
2. Expand the brackets.	$x^2 - 3x = -x - 1$
3. Rearrange the equation to get 0 on one side.	$x^2 - 2x + 1 = 0$
4. Factorise the quadratic and solve for x.	$(x - 1)^2 = 0$ so $\mathbf{x = 1}$

Quadratic Equations by Factorising

Week 2 Task 3

HM: 230, 231, 232, 233, 234

Exercise 2

Rearrange the following equations, then solve them by factorising.

Q1 a) $x^2 = x$ b) $x^2 + 2x = 3$ c) $10x - x^2 = 21$ d) $x^2 = 6x - 8$

e) $8x - x^2 = 12$ f) $3x^2 = 6x + 9$ g) $x^2 + 21x = 11 - x^2$ h) $4x^2 + 4x = 3$

i) $6x^2 + x = 1$ j) $6x^2 = 7x - 2$ k) $4x^2 + 1 = 4x$ l) $9x^2 + 25 = 30x$

Q2 a) $x(x - 2) = 8$ b) $x(x + 2) = 35$ c) $(x + 3)(x + 9) + 9 = 0$

d) $(x - 6)(x - 8) + 1 = 0$ e) $(x + 3)(x + 1) = 4x + 7$ f) $(2x + 1)(x - 1) = -16x - 8$

g) $(3x + 4)^2 = 7(3x + 4)$ h) $(3x - 4)(x - 2) - 5 = 0$ i) $2(4x + 1)(x - 1) + 3 = 0$

Q3 a) $x + 1 = \frac{6}{x}$ b) $x - 2 = \frac{4}{x + 1}$ c) $x + 2 = \frac{28}{x - 1}$

d) $2x + 1 = \frac{10}{4 - x}$ e) $3x - 1 = \frac{4}{2x + 1}$ f) $6x - 1 = \frac{4}{4x + 1}$

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

To write the quadratic $x^2 + bx + c$ in '**completed square form**', you need to make it look like this: $(x + p)^2 + q$. To get from the general form to the completed square form, follow this method:

- First, write out the **squared bracket**. The value of p is always half of b, i.e. $p = \frac{b}{2}$, so it's $\left(x + \frac{b}{2}\right)^2$.
- To find q, **expand the squared bracket** — you'll get $x^2 + bx + \left(\frac{b}{2}\right)^2$.
- Now **compare** this with the general form of the quadratic — the x^2 and x terms are the same, so add or subtract the **difference** between c and $\left(\frac{b}{2}\right)^2$ to make the **constant terms** the same.

To get from the completed square form to the general form just **expand the brackets** and **collect like terms**.

Completing the Square

Week 2 Task 4
HM: 235, 236, 237, 238, 239

Example 1

a) Write $x^2 + 6x - 5$ in completed square form.

1. $b = 6$, so $p = \frac{b}{2} = 3$. The squared bracket is:	$(x + 3)^2$
2. Expand the brackets.	$= x^2 + 6x + 9$
3. The constant term from the brackets is +9, but the constant you want is $c = -5$, so subtract 14 from the squared bracket.	$x^2 + 6x - 5 = x^2 + 6x + 9 - 14$ $= \mathbf{(x + 3)^2 - 14}$

b) Write $x^2 + 7x + 14$ in completed square form.

1. $b = 7$, so $p = \frac{b}{2} = \frac{7}{2}$. The squared bracket is:	$\left(x + \frac{7}{2}\right)^2$
2. Expand the brackets.	$= x^2 + 7x + \frac{49}{4}$
3. The constant from the brackets is $+\frac{49}{4} = 12\frac{1}{4}$, but the constant you want is $c = 14$, so add $1\frac{3}{4} = \frac{7}{4}$ to the squared bracket.	$x^2 + 7x + 14 = x^2 + 7x + \frac{49}{4} + \frac{7}{4}$ $= \boldsymbol{\left(x + \frac{7}{2}\right)^2 + \frac{7}{4}}$

Completing the Square

Week 2 Task 4
HM: 235, 236, 237, 238, 239

Exercise 1

Q1 Find the value of q in each of the following equations.

a) $x^2 - 4x + 7 = (x - 2)^2 + q$ b) $x^2 + 2x - 9 = (x + 1)^2 + q$ c) $x^2 + 4x + 2 = (x + 2)^2 + q$

Q2 Write the following quadratics in completed square form.

a) $x^2 + 2x + 6$ b) $x^2 - 2x + 4$ c) $x^2 - 2x - 10$ d) $x^2 - 12x + 100$

e) $x^2 + 12x + 44$ f) $x^2 + 14x$ g) $x^2 - 20x - 200$ h) $x^2 + 20x - 150$

Q3 Rewrite the quadratics below in the form $(x + p)^2 + q$.

a) $x^2 + 3x + 1$ b) $x^2 + 3x - 1$ c) $x^2 - 3x + 1$ d) $x^2 + 5x + 12$

e) $x^2 + 5x + 3$ f) $x^2 - 5x + 20$ g) $x^2 + 7x + 10$ h) $x^2 - 9x - 25$

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

If you have a quadratic of the form $ax^2 + bx + c$ ($\boldsymbol{a \neq 1}$), the method is pretty much the same, but the squared bracket is different. The completed square form in this case looks like $\boldsymbol{a(x + p)^2 + q}$.

This time $\boldsymbol{p = \frac{b}{2a}}$, so when you expand the squared bracket, you get $a\left(x^2 + 2\frac{b}{2a}x + \left(\frac{b}{2a}\right)^2\right) = ax^2 + bx + \frac{b^2}{4a}$.

The x^2 and x terms are the same, so just **compare** the **constant** terms like before to find q.

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

Example 2

Complete the square for the quadratic $3x^2 + 12x + 7$.

1. $b = 12$ and $a = 3$ so the bracket is: $3\left(x + \frac{12}{2 \times 3}\right)^2 = 3(x + 2)^2$
2. Expand the brackets — remember to multiply everything by the 3 at the front.
$= 3(x^2 + 4x + 4)$
$= 3x^2 + 12x + 12$
3. This quadratic has +12 as the constant but you want +7, so you need to subtract 5.
$3x^2 + 12x + 7$
$= 3x^2 + 12x + 12 - 5$
$= \boldsymbol{3(x + 2)^2 - 5}$

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

Exercise 2

Q1 Find the value of q in each of the following equations.

a) $2x^2 + x = 2\left(x+\frac{1}{4}\right)^2 + q$

b) $2x^2 + 20x + 500 = 2(x + 5)^2 + q$

c) $3x^2 + 4x + 25 = 3\left(x+\frac{2}{3}\right)^2 + q$

d) $4x^2 - 7x - 1 = 4\left(x-\frac{7}{8}\right)^2 + q$

Q2 Find the values of p and q in each of the following equations.

a) $2x^2 - 12x + 9 = 2(x + p)^2 + q$

b) $3x^2 - 5x - 1 = 3(x + p)^2 + q$

Q3 Write the following quadratics in completed square form.

a) $2x^2 + 8x + 81$

b) $3x^2 + 8x + 10$

c) $2x^2 - 2x + 3$

d) $5x^2 + 5x - 1$

e) $2x^2 - x + 1$

f) $3x^2 + 18x + 90$

g) $4x^2 + 13x + 9$

h) $2x^2 + 11x$

i) $-2x^2 + 10x - 2$

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

You can **solve** quadratics, including ones that **can't** easily be factorised, by **completing the square**. As with factorising, you should **rearrange** the equation into the form $\boldsymbol{ax^2 + bx + c = 0}$ before anything else.

Put $ax^2 + bx + c = 0$ into completed square form ($a(x + p)^2 + q$) and then **rearrange** to make x the subject: $\boldsymbol{x = -p \pm \sqrt{-\frac{q}{a}}}$. You can't take the square root of a negative number, so the quadratic only has solutions when $-\frac{q}{a} \geq 0$.

Tip: If $a \neq 1$ and you have 0 on one side, you could divide everything by a to potentially make completing the square simpler.

You might end up with a **surd** (see page 99) in your answer. If the question asks you to give an **exact answer**, leave the surd in — don't be tempted to use your calculator to find the decimal answer.

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

Example 3a

a) Find the exact solutions to the equation $x^2 - 4x + 1 = 0$ by completing the square.

1. $b = -4$, so $p = \frac{b}{2} = -2$. Expand $(x-2)^2$.

$$(x-2)^2 = x^2 - 4x + 4$$

2. Complete the square — you've got the constant +4 but need +1, so subtract 3.

$$x^2 - 4x + 1 = (x-2)^2 - 3$$

3. Use the completed square to rewrite and solve the original equation.

$$x^2 - 4x + 1 = 0$$
$$(x-2)^2 - 3 = 0$$
$$(x-2)^2 = 3$$

4. Don't forget there's a positive and negative square root.

$$x - 2 = \pm\sqrt{3}$$

5. For the exact solutions, leave in surd form.

$$\mathbf{x = 2 + \sqrt{3}} \text{ or } \mathbf{x = 2 - \sqrt{3}}$$

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

Example 3b

**b) Solve the equation $2x^2 - 8x + 3 = 0$ by completing the square.
Give your answers to 2 decimal places.**

1. $b = -8$ and $a = 2$. The squared bracket is:

$$2\left(x + \frac{-8}{2 \times 2}\right)^2 = 2(x-2)^2$$

2. Expand the brackets.

$$= 2x^2 - 8x + 8$$

3. The quadratic from the brackets has +8 but you want +3, so subtract 5 to complete the square.

$$2x^2 - 8x + 3 = 2x^2 - 8x + 8 - 5$$
$$= 2(x-2)^2 - 5$$

4. Use the completed square form to rewrite and solve the original equation. Get the squared bracket on its own and then take the square root of both sides (don't forget the ±). Then get x on its own.

$$2x^2 - 8x + 3 = 0,$$
$$\text{so } 2(x-2)^2 - 5 = 0$$
$$2(x-2)^2 = 5$$
$$(x-2)^2 = \frac{5}{2} \Rightarrow x - 2 = \pm\sqrt{\frac{5}{2}}$$
$$\text{so } x = 2 + \sqrt{\frac{5}{2}} \text{ or } x = 2 - \sqrt{\frac{5}{2}}$$

5. You're asked for the answers to 2 d.p., so use your calculator to evaluate the surds.

$$\mathbf{x = 3.58} \text{ (2 d.p.) or } \mathbf{x = 0.42} \text{ (2 d.p.)}$$

Completing the Square

Week 2 Task 4

HM: 235, 236, 237, 238, 239

Exercise 3

Q1 Find the exact solutions of the following equations by completing the square.

a) $x^2 - 2x - 4 = 0$ b) $x^2 + 4x + 3 = 0$ c) $x^2 + 6x - 4 = 0$

d) $x^2 + 8x + 4 = 0$ e) $x^2 - x - 1 = 0$ f) $x^2 - 11x + 25 = 0$

Q2 Solve the following equations by completing the square. Give your answers to 2 decimal places.

a) $x^2 + 6x + 4 = 0$ b) $x^2 - 2x - 5 = 0$ c) $x^2 + 6x - 3 = 0$

d) $x^2 + 8x + 8 = 0$ e) $x^2 - x - 10 = 0$ f) $x^2 - 5x + 3 = 0$

Q3 Find the exact solutions of the following equations by completing the square.

a) $3x^2 + 2x - 2 = 0$ b) $5x^2 + 2x = 10$ c) $4x^2 - 6x - 1 = 0$

d) $2x^2 = 12x - 5$ e) $3x^2 = 10 - 5x$ f) $10x^2 + 7x - 1 = 0$

Q4 Solve the following equations by completing the square. Give your answers to 2 decimal places.

a) $2x^2 + 2x - 3 = 0$ b) $3x^2 + 2x - 7 = 0$ c) $4x^2 + 8x = 11$

d) $2x^2 - 16x - 19 = 0$ e) $6x^2 = 1 - 3x$ f) $3x^2 + 9x - 7 = 0$

The Quadratic Formula

Week 2 Task 5

HM: 240, 241, 242

The **quadratic formula** is a quick way to work out all the possible solutions to a quadratic equation. To use it, first make sure the equation is in the form $\boldsymbol{ax^2 + bx + c = 0}$ and then **substitute** a, b and c into this formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To **derive** the quadratic formula, solve $ax^2 + bx + c = 0$ by '**completing the square**':

$$ax^2 + bx + c = 0 \Rightarrow a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0 \Rightarrow a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$\Rightarrow a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a} \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \Rightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$ which is the **quadratic formula**.

The formula might seem pretty simple to use, but **watch out** for these things:

- **Minus signs** can cause confusion — if b is negative then $-b$ will be positive, and if one of a or c is negative then $-4ac$ will be positive.
- Divide **everything** on top by $2a$, not just the square root.
- Don't forget the ± **sign** — otherwise you won't get **both solutions**.

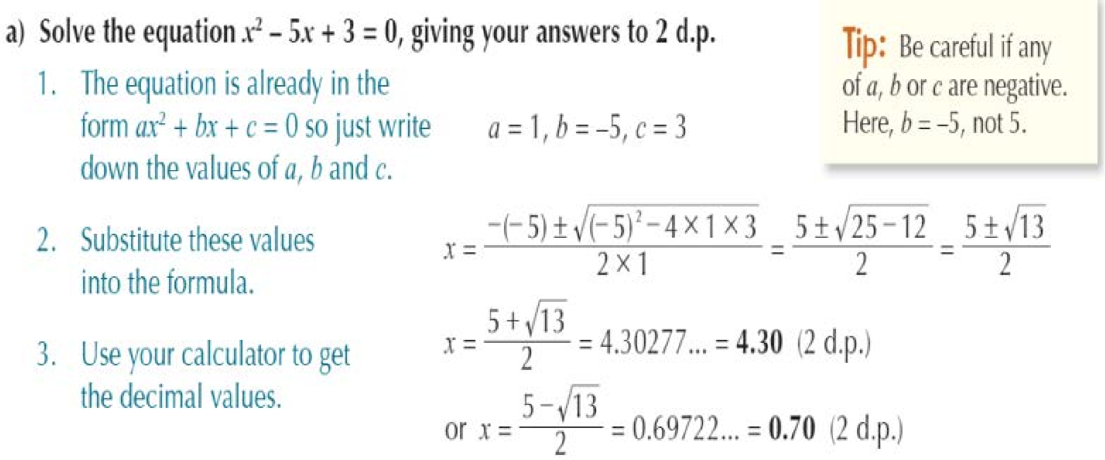
Tip: If the question mentions exact answers or decimal places then the quadratic probably won't factorise easily, so use the formula (although you could use completing the square in these cases too).

The Quadratic Formula

Week 2 Task 5

HM: 240, 241, 242

Example 1a



a) Solve the equation $x^2 - 5x + 3 = 0$, giving your answers to 2 d.p.

1. The equation is already in the form $ax^2 + bx + c = 0$ so just write down the values of a, b and c. $a = 1, b = -5, c = 3$
2. Substitute these values into the formula. $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 3}}{2 \times 1} = \frac{5 \pm \sqrt{25 - 12}}{2} = \frac{5 \pm \sqrt{13}}{2}$
3. Use your calculator to get the decimal values. $x = \frac{5 + \sqrt{13}}{2} = 4.30277... = \mathbf{4.30}$ (2 d.p.) or $x = \frac{5 - \sqrt{13}}{2} = 0.69722... = \mathbf{0.70}$ (2 d.p.)

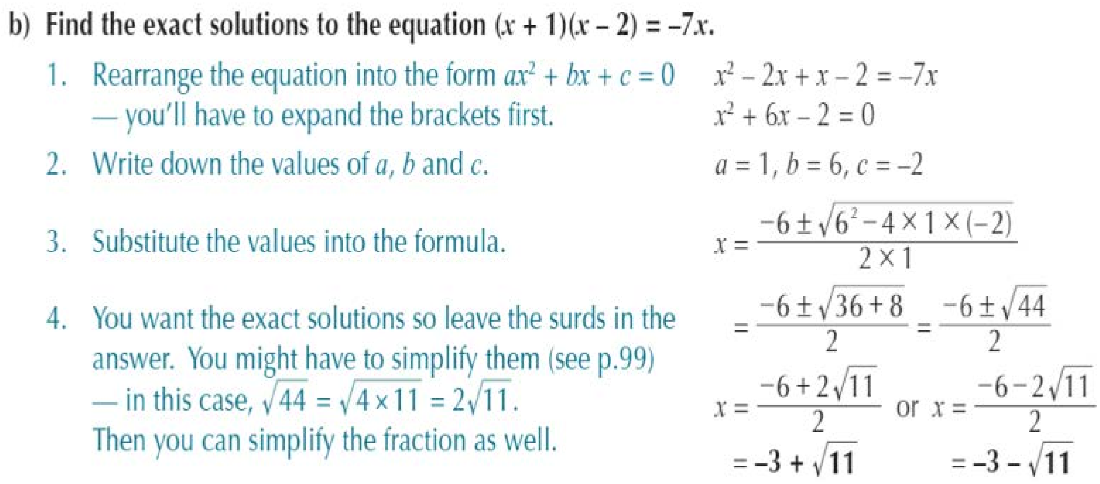
Tip: Be careful if any of a, b or c are negative. Here, $b = -5$, not 5.

The Quadratic Formula

Week 2 Task 5

HM: 240, 241, 242

Example 1b



b) Find the exact solutions to the equation $(x + 1)(x - 2) = -7x$.

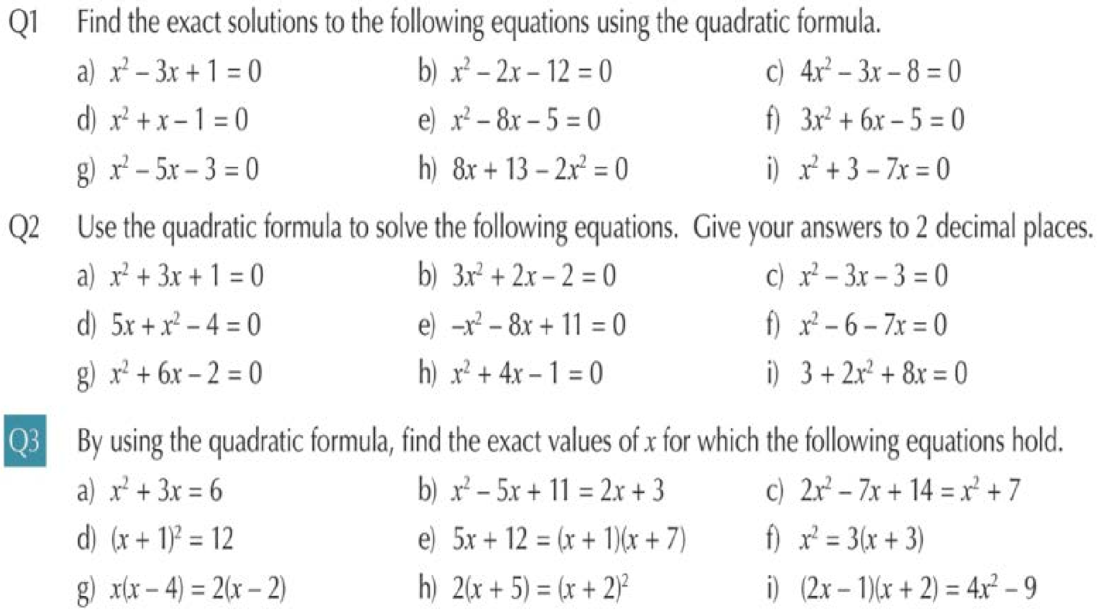
1. Rearrange the equation into the form $ax^2 + bx + c = 0$ — you'll have to expand the brackets first.
   $x^2 - 2x + x - 2 = -7x$
   $x^2 + 6x - 2 = 0$
2. Write down the values of a, b and c. $a = 1, b = 6, c = -2$
3. Substitute the values into the formula. $x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times (-2)}}{2 \times 1}$
4. You want the exact solutions so leave the surds in the answer. You might have to simplify them (see p.99) — in this case, $\sqrt{44} = \sqrt{4 \times 11} = 2\sqrt{11}$. Then you can simplify the fraction as well.
   $= \frac{-6 \pm \sqrt{36 + 8}}{2} = \frac{-6 \pm \sqrt{44}}{2}$
   $x = \frac{-6 + 2\sqrt{11}}{2}$ or $x = \frac{-6 - 2\sqrt{11}}{2}$
   $= \mathbf{-3 + \sqrt{11}}$ $= \mathbf{-3 - \sqrt{11}}$

The Quadratic Formula

Week 2 Task 5

HM: 240, 241, 242

Exercise 1



Q1 Find the exact solutions to the following equations using the quadratic formula.

a) $x^2 - 3x + 1 = 0$
b) $x^2 - 2x - 12 = 0$
c) $4x^2 - 3x - 8 = 0$
d) $x^2 + x - 1 = 0$
e) $x^2 - 8x - 5 = 0$
f) $3x^2 + 6x - 5 = 0$
g) $x^2 - 5x - 3 = 0$
h) $8x + 13 - 2x^2 = 0$
i) $x^2 + 3 - 7x = 0$

Q2 Use the quadratic formula to solve the following equations. Give your answers to 2 decimal places.

a) $x^2 + 3x + 1 = 0$
b) $3x^2 + 2x - 2 = 0$
c) $x^2 - 3x - 3 = 0$
d) $5x + x^2 - 4 = 0$
e) $-x^2 - 8x + 11 = 0$
f) $x^2 - 6 - 7x = 0$
g) $x^2 + 6x - 2 = 0$
h) $x^2 + 4x - 1 = 0$
i) $3 + 2x^2 + 8x = 0$

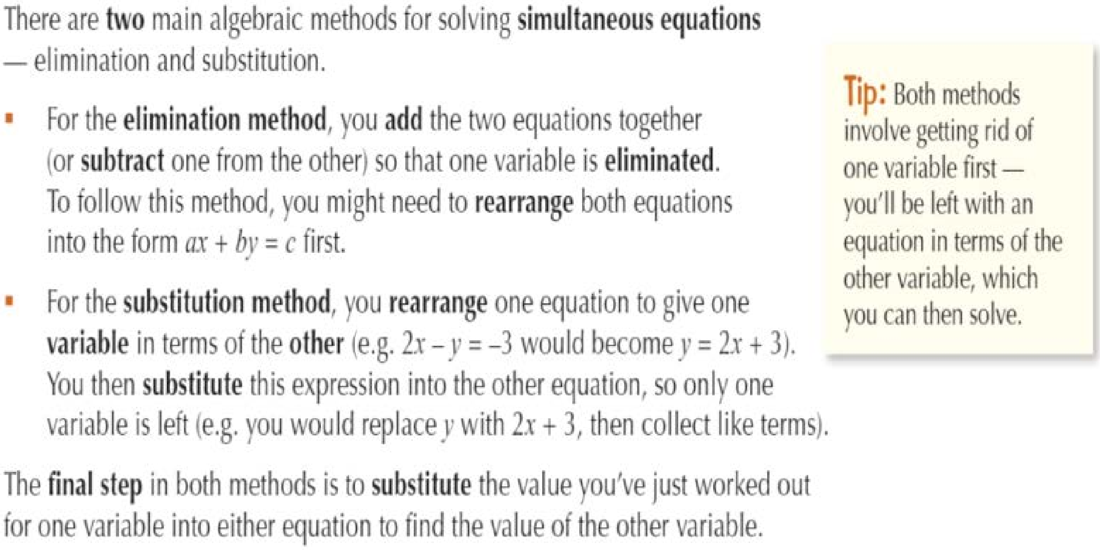
Q3 By using the quadratic formula, find the exact values of x for which the following equations hold.

a) $x^2 + 3x = 6$
b) $x^2 - 5x + 11 = 2x + 3$
c) $2x^2 - 7x + 14 = x^2 + 7$
d) $(x + 1)^2 = 12$
e) $5x + 12 = (x + 1)(x + 7)$
f) $x^2 = 3(x + 3)$
g) $x(x - 4) = 2(x - 2)$
h) $2(x + 5) = (x + 2)^2$
i) $(2x - 1)(x + 2) = 4x^2 - 9$

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194



There are **two** main algebraic methods for solving **simultaneous equations** — elimination and substitution.

- For the **elimination method**, you **add** the two equations together (or **subtract** one from the other) so that one variable is **eliminated**. To follow this method, you might need to **rearrange** both equations into the form $ax + by = c$ first.
- For the **substitution method**, you **rearrange** one equation to give one **variable** in terms of the **other** (e.g. $2x - y = -3$ would become $y = 2x + 3$). You then **substitute** this expression into the other equation, so only one variable is left (e.g. you would replace y with $2x + 3$, then collect like terms).

The **final step** in both methods is to **substitute** the value you've just worked out for one variable into either equation to find the value of the other variable.

Tip: Both methods involve getting rid of one variable first — you'll be left with an equation in terms of the other variable, which you can then solve.

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194

Example 1

Solve the simultaneous equations: **(1)** $11 - x = y$
(2) $3y = x - 7$

1.	Rearrange the equations so that they're both in the form $ax + by = c$.	(1) $x + y = 11$ (2) $x - 3y = 7$
2.	You've got $+x$ in both equations so subtract equation (2) from equation (1) to eliminate x.	$\begin{array}{r} x + y = 11 \\ -(x - 3y = 7) \\ \hline 4y = 4 \end{array}$
3.	Solve the resulting equation for y.	$y = 1$
4.	Put $y = 1$ into one of the original equations and solve for x.	$x + 1 = 11$ $x = 10$
5.	Use the other equation to check the answer.	$x - 3y = 10 - 3(1) = 7$ ✔ So $\boldsymbol{x = 10,\ y = 1}$

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194

Example 2

Solve the simultaneous equations: **(1)** $7x - y = 16$
(2) $y - 2 = x$

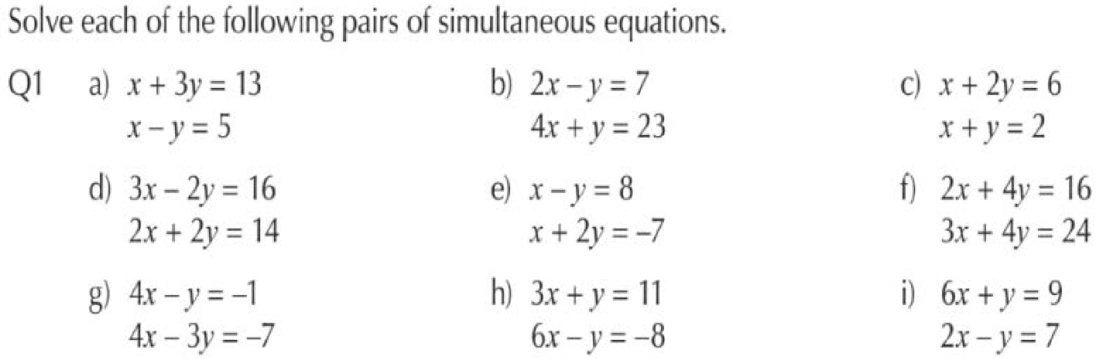
1.	You've got y in both equations so rearrange equation (2) to get y on its own.	$y = x + 2$
2.	Substitute $y = x + 2$ into equation (1).	$7x - (x + 2) = 16$
3.	Solve for x.	$6x - 2 = 16$ $6x = 18$ $x = 3$
4.	Put $x = 3$ into one of the original equations and solve for y.	$y - 2 = 3$ $y = 5$
5.	Use the other equation to check the answer.	$7x - y = 7(3) - 5 = 16$ ✔ So $\boldsymbol{x = 3,\ y = 5}$

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194

Exercise 1



Solve each of the following pairs of simultaneous equations.

Q1 a) $x + 3y = 13$
$x - y = 5$

b) $2x - y = 7$
$4x + y = 23$

c) $x + 2y = 6$
$x + y = 2$

d) $3x - 2y = 16$
$2x + 2y = 14$

e) $x - y = 8$
$x + 2y = -7$

f) $2x + 4y = 16$
$3x + 4y = 24$

g) $4x - y = -1$
$4x - 3y = -7$

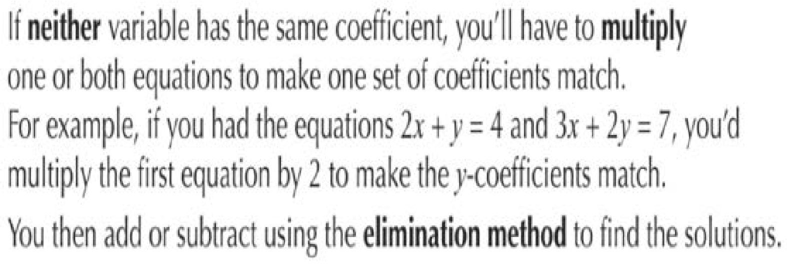
h) $3x + y = 11$
$6x - y = -8$

i) $6x + y = 9$
$2x - y = 7$

Simultaneous Linear Equations

Week 2 Task 6

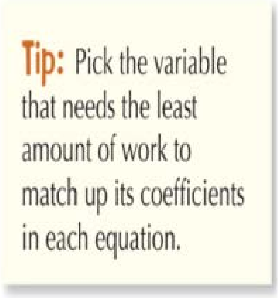
HM: 191, 192, 193, 194



If **neither** variable has the same coefficient, you'll have to **multiply** one or both equations to make one set of coefficients match.

For example, if you had the equations $2x + y = 4$ and $3x + 2y = 7$, you'd multiply the first equation by 2 to make the y-coefficients match.

You then add or subtract using the **elimination method** to find the solutions.



Tip: Pick the variable that needs the least amount of work to match up its coefficients in each equation.

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194

Example 3

Solve the simultaneous equations: **(1)** $5x - 4y = 23$
(2) $2x + 6y = -25$

> **Tip:** You can label the new equations as (3) and (4) to help you keep track of things. Then Step 2 in this example is (3) + (4).

1. Multiply equation (1) by 3 and equation (2) by 2 to match the y-coefficients.
2. Add the resulting equations to eliminate y and solve the equation for x.

$$\begin{array}{lrcl} 3 \times (1): & 15x - 12y & = & 69 \\ 2 \times (2): + & 4x + 12y & = & -50 \\ \hline & 19x & = & 19 \\ & x & = & 1 \end{array}$$

3. Put $x = 1$ into one of the original equations and solve for y.

$5(1) - 4y = 23 \Rightarrow 5 - 4y = 23$
$\Rightarrow -4y = 18 \Rightarrow y = -4.5$

4. Use the other equation to check the answer.

$2x + 6y = 2(1) + 6(-4.5) = 2 - 27 = -25$ ✓
So $\mathbf{x = 1}$**,** $\mathbf{y = -4.5}$

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194

Exercise 2

Solve each of the following pairs of simultaneous equations.

Q1 a) $3x + 2y = 16$
$2x + y = 9$

b) $4x + 3y = 16$
$5x - y = 1$

c) $4x - y = 22$
$3x + 4y = 26$

d) $2x + 3y = 10$
$x - y = 5$

e) $4x - 2y = 8$
$x - 3y = -3$

f) $3e - 5r = 17$
$9e + 2r = -17$

Q2 a) $3x - 2y = 8$
$5x - 3y = 14$

b) $4p + 3q = 17$
$3p - 4q = 19$

c) $4u + 7v = 15$
$5u - 2v = 8$

d) $2c + 6d = 19$
$3c + 8d = 28$

e) $3r - 4s = -22$
$8r + 3s = -4$

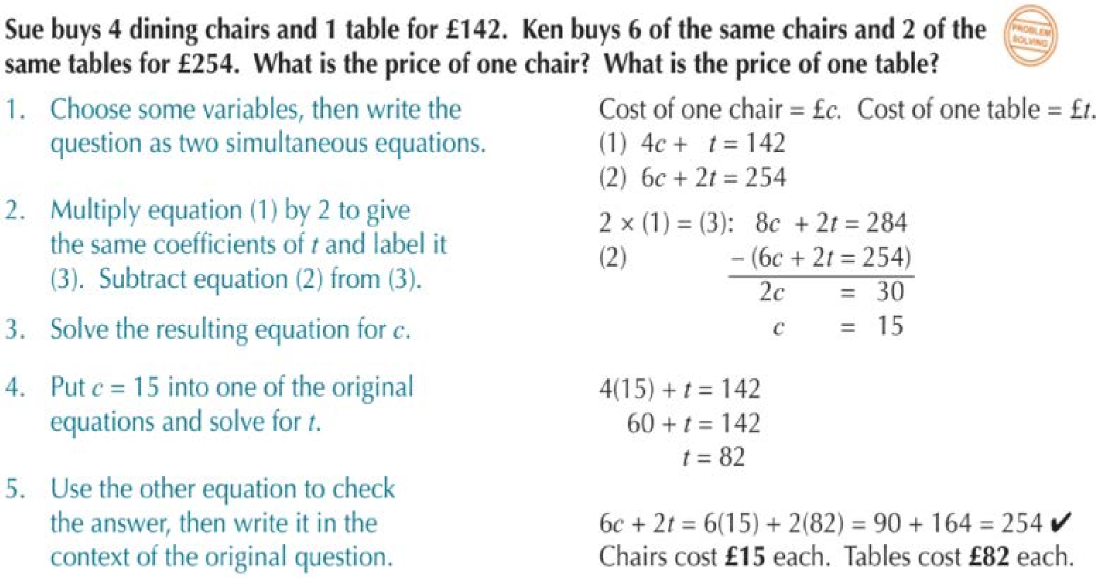
f) $3m + 5n = 14$
$7m + 2n = 23$

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194

Example 4



Sue buys 4 dining chairs and 1 table for £142. Ken buys 6 of the same chairs and 2 of the same tables for £254. What is the price of one chair? What is the price of one table?

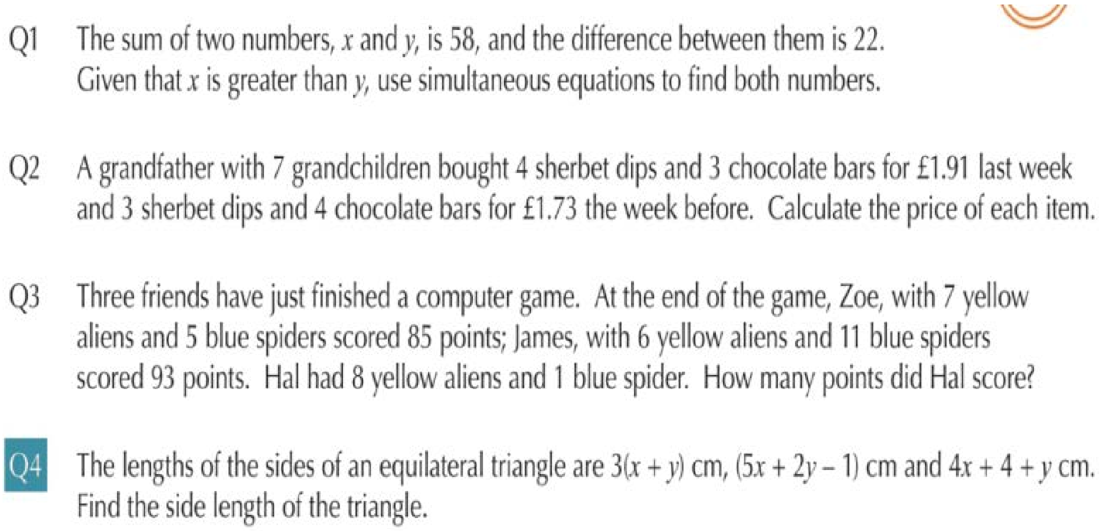
1. Choose some variables, then write the question as two simultaneous equations.	Cost of one chair = £c. Cost of one table = £t. (1) $4c + t = 142$ (2) $6c + 2t = 254$
2. Multiply equation (1) by 2 to give the same coefficients of t and label it (3). Subtract equation (2) from (3).	$2 \times (1) = (3)$: $8c + 2t = 284$ (2) $-(6c + 2t = 254)$ $2c = 30$
3. Solve the resulting equation for c.	$c = 15$
4. Put $c = 15$ into one of the original equations and solve for t.	$4(15) + t = 142$ $60 + t = 142$ $t = 82$
5. Use the other equation to check the answer, then write it in the context of the original question.	$6c + 2t = 6(15) + 2(82) = 90 + 164 = 254$ ✔ Chairs cost **£15** each. Tables cost **£82** each.

Simultaneous Linear Equations

Week 2 Task 6

HM: 191, 192, 193, 194

Exercise 3



Q1 The sum of two numbers, x and y, is 58, and the difference between them is 22. Given that x is greater than y, use simultaneous equations to find both numbers.

Q2 A grandfather with 7 grandchildren bought 4 sherbet dips and 3 chocolate bars for £1.91 last week and 3 sherbet dips and 4 chocolate bars for £1.73 the week before. Calculate the price of each item.

Q3 Three friends have just finished a computer game. At the end of the game, Zoe, with 7 yellow aliens and 5 blue spiders scored 85 points; James, with 6 yellow aliens and 11 blue spiders scored 93 points. Hal had 8 yellow aliens and 1 blue spider. How many points did Hal score?

Q4 The lengths of the sides of an equilateral triangle are $3(x + y)$ cm, $(5x + 2y - 1)$ cm and $4x + 4 + y$ cm. Find the side length of the triangle.

Simultaneous Linear & QuadraticEquations

Week 2 Task 7

HM: 246

To solve simultaneous equations with one **linear equation** and one **quadratic** equation, use the **substitution method**.

- First **rearrange** the **linear equation** to get it in terms of **one variable** (e.g. $2x - y = -3$ would become $y = 2x + 3$).
- **Substitute** this into the **quadratic equation** — you'll be left with a quadratic equation in one variable.
- **Solve** the resulting quadratic equation — you'll probably get **2 solutions**.
- **Substitute** each of these values into one of the original equations to find the values of the other variable — the **linear equation** is often the easiest to use.

Tip: If there's a variable with a coefficient of 1, it's usually best to make that the subject so it can be easily substituted into the other equation.

The **solutions** to these pairs of simultaneous equations correspond to the **points on a graph** where a straight line and a quadratic curve **cross**. If the straight line crosses the curve **twice** there will be **2 pairs** of solutions to the simultaneous equations. The line could also cross the curve only **once** or **not at all**.

Simultaneous Linear & QuadraticEquations

Week 2 Task 7

HM: 246

Example 1

Solve the simultaneous equations: **(1)** $x + y = 5$
(2) $y = x^2 - 3x - 30$

1. Rearrange equation (1) to get y on its own. $y = 5 - x$
2. Substitute $y = 5 - x$ into $y = x^2 - 3x - 30$. $5 - x = x^2 - 3x - 30$
3. Rearrange to get zero on the right hand side. $x^2 - 3x + x - 30 - 5 = 0$
   $x^2 - 2x - 35 = 0$
4. Solve the quadratic. $(x + 5)(x - 7) = 0$
   $x + 5 = 0$ or $x - 7 = 0$
   so $x = -5$ or $x = 7$
5. Substitute into equation (1) to find a y-value for each value of x. If $x = -5$, then $-5 + y = 5$, so $y = 10$.
   If $x = 7$, then $7 + y = 5$, so $y = -2$.
6. Write the solutions in pairs. $\boldsymbol{x = -5, y = 10}$ **and** $\boldsymbol{x = 7, y = 2}$

Tip: If you'd rearranged (1) to make x the subject, you'd have to do a bit more work at step 2.

Simultaneous Linear & QuadraticEquations

Week 2 Task 7

HM: 246

Exercise 1

Q1 Find the solutions to each of the following pairs of simultaneous equations.

a) $y = x^2 - 4x + 8$
$y = 2x$

b) $y = x^2 - x - 1$
$3x = 2 - y$

c) $y = x^2 - 4x - 28$
$y = 3x + 2$

d) $y = x^2 + 3x - 2$
$y + x = 3$

e) $y = x^2 - 4x + 2$
$y = 2x - 6$

f) $y = x^2 - 2x - 3$
$y = 3x + 11$

g) $y = x^2 - x - 5$
$y = 2x + 5$

h) $y = x^2 - 4x + 8$
$4 = y - x$

i) $y = 2x^2 + x - 2$
$y = 8x - 5$

Simultaneous Linear & QuadraticEquations

Week 2 Task 7

HM: 246

Exercise 1

Q2 The line $2y = x + 3$ and the curve $y = x^2 - 2x - 2$ cross at points M and N. Find the coordinates of M and N by solving the simultaneous equations.

Q3 Find where the line $y = 4 - 3x$ crosses the curve $y = 6x^2 + 10x - 1$ by solving the equations simultaneously.

Q4 Use simultaneous equations to find the coordinates where the line $y = 5x$ meets the curve $y = x^2 + 3x + 1$. What can you say about the line and the curve?

Q5 Solve these equations simultaneously: $x - 4y = 2$ and $y^2 + xy = 0$

Q6 Solve these pairs of simultaneous equations.

a) $x + y = 7$
$x^2 - xy = 4$

b) $x + y = 5$
$x + xy + 2y^2 = 2$

c) $x + y = 2$
$y^2 - x = 0$

d) $x - y = 4$
$x^2 + y = 2$

e) $x + y = 4$
$x^2 + 3xy = 16$

f) $4y + x = 10$
$xy + x = -8$

Simultaneous Linear & QuadraticEquations

Week 2 Task 7

HM: 246

Example 2

Solve the simultaneous equations: **(1)** $x^2 + y^2 = 10$
(2) $2x + y = 5$

1. Rearrange equation (2) to get y on its own.

$$y = 5 - 2x$$

2. Substitute $y = 5 - 2x$ into $x^2 + y^2 = 10$.

$$x^2 + (5 - 2x)^2 = 10$$
$$x^2 + 25 - 20x + 4x^2 = 10$$

Tip: Equation (1) is the equation of a circle (see p.203).

3. Rearrange to get zero on the right hand side.

$$5x^2 - 20x + 15 = 0$$
$$x^2 - 4x + 3 = 0$$

4. Solve the quadratic.

$$(x - 1)(x - 3) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$\text{so} \;\; x = 1 \quad \text{or} \quad x = 3$$

5. Substitute into equation (2) to find a y-value for each value of x.

If $x = 1$, then $2(1) + y = 5$, so $y = 3$
If $x = 3$, then $2(3) + y = 5$, so $y = -1$

6. Write the solutions in pairs.

$\mathbf{x = 1, y = 3}$ **and** $\mathbf{x = 3, y = -1}$

Simultaneous Linear & QuadraticEquations

Week 2 Task 7

HM: 246

Exercise 2

Q1 Solve these pairs of simultaneous equations.

a) $2x + y = 3$
$y^2 - x^2 = 0$

b) $3x + y = 4$
$x^2 + 3xy + y^2 = -16$

c) $x - y = -4$
$x^2 + y^2 - x = 20$

Q2 The equations $x - y = -3$ and $3x^2 + 7x + y^2 = 21$ are plotted on a graph.

a) Show that at the points of intersection, $4x^2 + 13x - 12 = 0$.

b) Find the exact values of x when $4x^2 + 13x - 12 = 0$.

c) Find the exact coordinates of the points of intersection.

Q3 Find the exact coordinates where the graphs of the following pairs of equations intersect.

a) $x = 3y + 4$
$x^2 + y^2 = 34$

b) $2x + 2y = 1$
$x^2 + y^2 = 1$

c) $\sqrt{5}y - x = 6$
$x^2 + y^2 = 36$

Week 2 Assessment

Question 1

The minimum velocity required for a rocket to leave a planet can be found using the formula $V = \sqrt{\frac{2GM}{r}}$. Make M the subject of this formula. Show your working.

[3 marks]

Week 2 Assessment

Question 2

Solve the following equations:

a) $\frac{2(x+3)}{13} = 1$

[2 marks]

b) $\frac{x-10}{10} = \frac{10-x}{3}$

[2 marks]

c) $3(x^2 + 7) = 4(x^2 - 1)$

[2 marks]

Week 2 Assessment

Question 3

a) Write the quadratic $x^2 + 8x + 12$ in the form $(x + a)^2 + b$.

[2 marks]

b) Hence, or otherwise, solve the equation $x^2 + 8x + 12 = 0$.

[2 marks]

Week 2 Assessment

Question 4

a) Write the quadratic $x^2 + 8x + 12$ in the form $(x + a)^2 + b$.

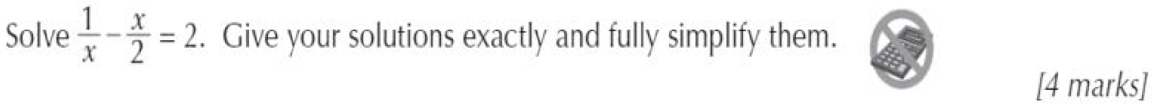
[2 marks]

b) Hence, or otherwise, solve the equation $x^2 + 8x + 12 = 0$.

[2 marks]

Week 2 Assessment

Question 5

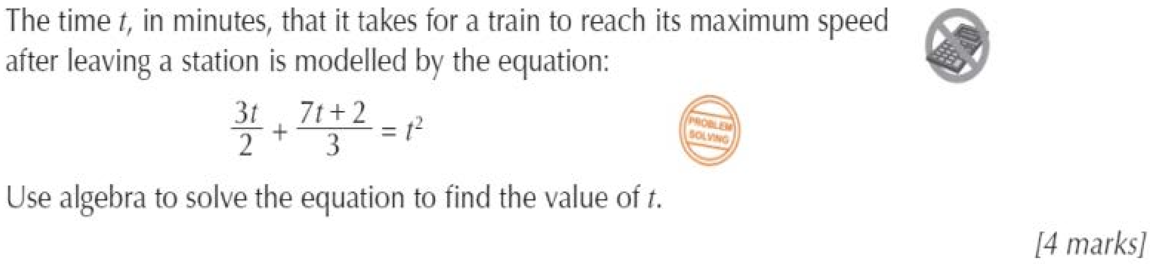


Solve $\frac{1}{x} - \frac{x}{2} = 2$. Give your solutions exactly and fully simplify them.

[4 marks]

Week 2 Assessment

Question 6



The time t, in minutes, that it takes for a train to reach its maximum speed after leaving a station is modelled by the equation:

$$\frac{3t}{2} + \frac{7t+2}{3} = t^2$$

Use algebra to solve the equation to find the value of t.

[4 marks]

Week 2 Assessment

Question 7

Solve the simultaneous equations.

$$7m + 2n = 23$$
$$3m + 5n = 14$$

[3 marks]

Week 2 Assessment

Question 8

The equation of line A is $y = 2x - 2$, and the equation of line B is $5y = 20 - 2x$.
Find the coordinates of the point where lines A and B cross.

[3 marks]

Week 2 Assessment

Question 9

Solve the simultaneous equations.

$$y = 2x + 2$$
$$x^2 + y^2 = 8$$

[5 marks]

Week 2 Assessment

Question 10

Find the coordinates of the points where the curve $y = x^2$ and the line $y = 6x - 8$ intersect.

[4 marks]

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Year 11 Transition Tasks to A Level Mathematics

Week 3 Tasks

Week 3 Tasks

1. Horizontal & Vertical Lines HM: 205

2. Straight Line Graphs HM: 206

3. Gradients HM: 201, 202, 203

4. Equations of Linear Graphs HM: 207, 208, 209, 210, 211, 212, 213

5. Parallel & Perpendicular Lines HM: 214, 215, 216

6. Line Segments HM: 200

7. Quadratic Graphs HM: 251, 252, 253, 254, 255, 256, 257

Horizontal & Vertical Lines

Week 3 Task 1

HM: 205

All **horizontal** lines have the equation $y = a$ (where a is a number),
since every point on the same horizontal line has the same y-coordinate (a).
To **draw** the line $y = a$, draw a horizontal line that passes through a on the **y-axis**.

All **vertical** lines have the equation $x = b$ (where b is a number),
since every point on the same vertical line has the same x-coordinate (b).
To **draw** the line $x = b$, draw a vertical line that passes through b on the **x-axis**.

The equation of the **x-axis** is $y = 0$ and the equation of the **y-axis** is $x = 0$.

Horizontal & Vertical Lines

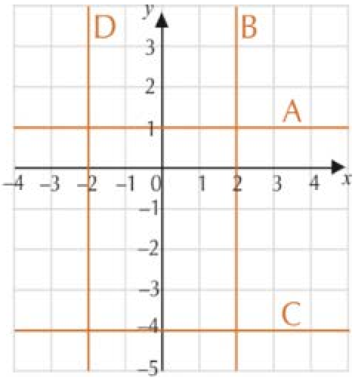
Week 3 Task 1

HM: 205

Example 1

Write down the equations of the lines marked A-D.

1. Every point on the line marked A has y-coordinate 1. — A is the line $y = 1$
2. Every point on the line marked B has x-coordinate 2. — B is the line $x = 2$
3. Every point on the line marked C has y-coordinate –4. — C is the line $y = -4$
4. Every point on the line marked D has x-coordinate –2. — D is the line $x = -2$

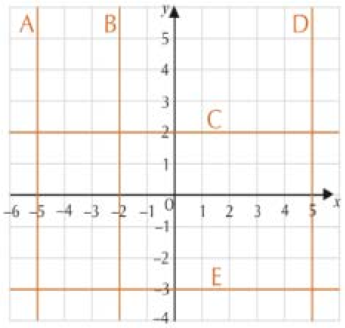


Horizontal & Vertical Lines

Week 3 Task 1

HM: 205

Exercise 1



Q1 Write down the equations of each of the lines labelled A to E on the right.

Q2 Draw a set of coordinate axes and plot the graphs with the following equations.

a) $y = 3$	b) $y = -6$	c) $y = -1$
d) $x = 2$	e) $x = 4$	f) $x = -6$

Q3 Write down the equation of each of the following.

a) The line which is parallel to the x-axis, and which passes through the point (4, 8).

b) The line which is parallel to the y-axis, and which passes through the point (–2, –6).

c) The line which is parallel to the line $x = 4$, and which passes through the point (1, 1).

d) The line which is parallel to the line $y = -5$, and which passes through the point (0, 6).

Q4 Write down the coordinates of the points where the following pairs of lines intersect.

a) $x = 8$ and $y = -11$ b) $x = -5$ and $y = -13$ c) $x = -\frac{6}{11}$ and $y = -500$

Straight Line Graphs

Week 3 Task 2

HM: 206

The equation of a straight line which **isn't** horizontal or vertical contains **both x and y** — e.g. $y = 2x + 4$. If an equation **only** contains x and y terms (e.g. $y = 5x$), then the line passes through the **origin (0, 0)**.

There are a couple of different methods you can use to draw these straight-line graphs:

- Make a **table of values** — find the values of y for different values of x, plot the points and join with a straight line. You only have to plot two points to be able to sketch the graph — but it's often useful to plot more than two, in case one of the points you plot is incorrect.
- Find the **value of x** when **$y = 0$** and the **value of y** when **$x = 0$**. Plot these two points and join with a straight line. Both points should lie on the axes. **Extend** your line to cover the range of x-values required (usually specified in the question).

Straight Line Graphs

Week 3 Task 2

HM: 206

Example 2

a) **Complete the table to show the value of $y = 2x + 1$ for values of x from 0 to 5.**

x	0	1	2	3	4	5
y						

Use the equation $y = 2x + 1$ to find the y-value corresponding to each value of x.

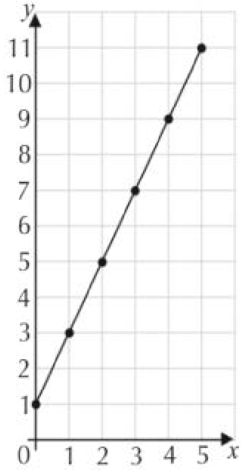
x	0	1	2	3	4	5
y	$2 \times 0 + 1 = \mathbf{1}$	$2 \times 1 + 1 = \mathbf{3}$	$2 \times 2 + 1 = \mathbf{5}$	$2 \times 3 + 1 = \mathbf{7}$	$2 \times 4 + 1 = \mathbf{9}$	$2 \times 5 + 1 = \mathbf{11}$

b) **Plot the points from the table, and hence draw the graph of $y = 2x + 1$ for values of x from 0 to 5.**

1. Use your table to find the coordinates to plot — just read off the x- and y-values from each column.

   The points to plot are (0, 1), (1, 3), (2, 5), (3, 7), (4, 9) and (5, 11).

2. Plot each point on the grid, then join them up with a straight line.



Straight Line Graphs

Week 3 Task 2

HM: 206

Example 3

Draw the graph of $y = 4 - 2x$ for $-1 \le x \le 3$.

1. Put $x = 0$ into the equation to find the value of y — this is where it crosses the y-axis.

   When $x = 0$, $y = 4 - 2(0) = \mathbf{4}$.

2. Put $y = 0$ into the equation to find the value of x — this is where it crosses the x-axis.

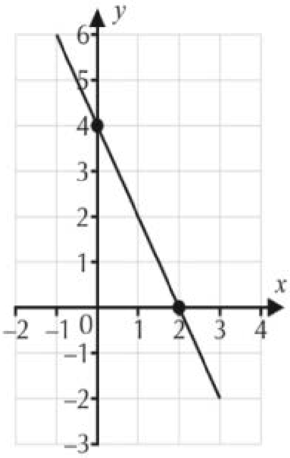
   When $y = 0$, $0 = 4 - 2x$

   $2x = 4$

   $x = \mathbf{2}$

   So the graph crosses the axes at **(0, 4)** and **(2, 0)**.

3. Mark the points (0, 4) and (2, 0) on your graph and draw a straight line passing through them. Make sure you extend it to cover the whole range of x-values asked for in the question.



Straight Line Graphs

Week 3 Task 2
HM: 206

Exercise 2

Q1 a) Copy and complete this table to show the value of $y = 2x$ and the coordinates of points on the line $y = 2x$ for values of x from -2 to 2.

x	-2	-1	0	1	2
y					
Coordinates					

b) Draw a set of axes with x-values from -5 to 5 and y-values from -10 to 10.
Plot the coordinates from your table.

c) Join up the points to draw the graph with equation $y = 2x$ for values of x from -2 to 2.

d) Use a ruler to extend your line to show the graph of $y = 2x$ for values of x from -5 to 5.

e) Use your graph to fill in the missing coordinates of these points on the line:

(i) (4, ☐) (ii) (-3, ☐) (iii) (☐, -10)

Q2 a) Copy and complete the table to show the value of $y = 8 - x$ and the coordinates of points on the line $y = 8 - x$ for values of x from 0 to 4.

x	0	1	2	3	4
y					
Coordinates					

b) Draw a set of axes with x-values from -5 to 5 and y-values from 0 to 13.
Plot the coordinates from your table.

c) Join up the points to draw the graph of $y = 8 - x$ for values of x from 0 to 4.
Use a ruler to extend your line to show the graph of $y = 8 - x$ for values of x from -5 to 5.

Straight Line Graphs

Week 3 Task 2
HM: 206

Exercise 2

Q3 For each of the following equations draw a graph for values of x from -5 to 5.

a) $y = -4x$
b) $y = \frac{x}{2}$
c) $y = 2x + 5$
d) $y = 4 - x$
e) $y = 8 - 3x$
f) $y = \frac{x}{4} + 1$

Q4 Draw a graph of the following equations for the given range of x-values.

a) $y = x + 7$ for $-7 \leq x \leq 0$
b) $y = -2x + 8$ for $0 \leq x \leq 5$
c) $y = 1.5x$ for $-5 \leq x \leq 5$
d) $y = 0.5x + 2$ for $-2 \leq x \leq 4$

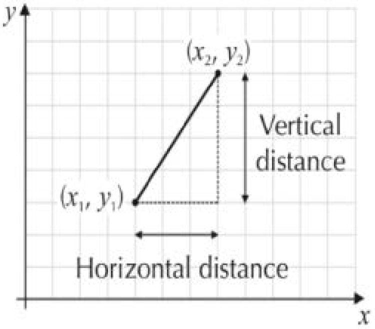
Gradients

Week 3 Task 3

HM: 201, 202, 203

To find the gradient of a line, divide the '**vertical distance**' (the change in the y-coordinates) between two points on the line by the '**horizontal distance**' (the change in the x-coordinates) between those points.

$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Make sure you subtract the x-coordinates and y-coordinates in the **same order** — i.e. if you do $y_2 - y_1$ on the numerator, you must do $x_2 - x_1$ on the denominator.

A line sloping **upwards** from left to right has a **positive gradient**.
A line sloping **downwards** from left to right has a **negative gradient**.

Regardless of the type of graph, the gradient always means '**y-axis units per x-axis units**'.

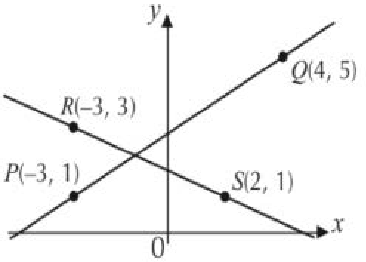
Gradients

Week 3 Task 3

HM: 201, 202, 203

Example 1

Find the gradient of the line that passes through:
a) points $P(-3, 1)$ and $Q(4, 5)$ b) points $R(-3, 3)$ and $S(2, 1)$



1. Call the coordinates of P (x_P, y_P), the coordinates of Q (x_Q, y_Q), the coordinates of R (x_R, y_R) and the coordinates of S (x_S, y_S).
2. Use the formula for the gradient.
3. The line through P and Q slopes upwards from left to right, so you should get a positive answer.
4. The line through R and S slopes downward from left to right, so you should get a negative answer.

a) Gradient of $PQ = \frac{y_Q - y_P}{x_Q - x_P} = \frac{5 - 1}{4 - (-3)} = \frac{4}{7}$

b) Gradient of $RS = \frac{y_R - y_S}{x_R - x_S} = \frac{3 - 1}{(-3) - 2} = -\frac{2}{5}$

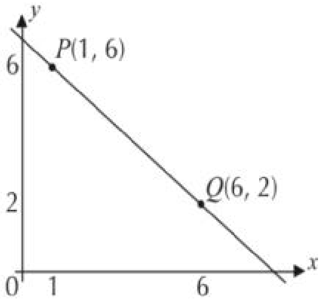
Gradients

Week 3 Task 3

HM: 201, 202, 203

Exercise 1

Q1 Points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are plotted on this graph.



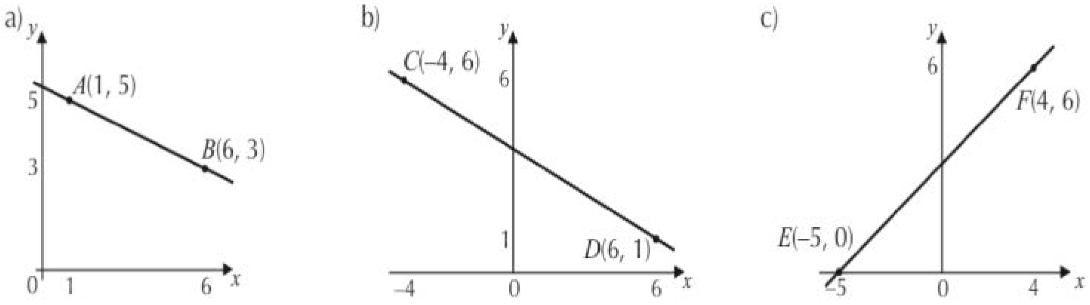
a) Without doing any calculations, state whether the gradient of the line containing P and Q is positive or negative.

b) Calculate the vertical distance $y_2 - y_1$ between P and Q.

c) Calculate the horizontal distance $x_2 - x_1$ between P and Q.

d) Find the gradient of the line containing P and Q.

Q2 Use the points shown to find the gradient of each of the following lines.



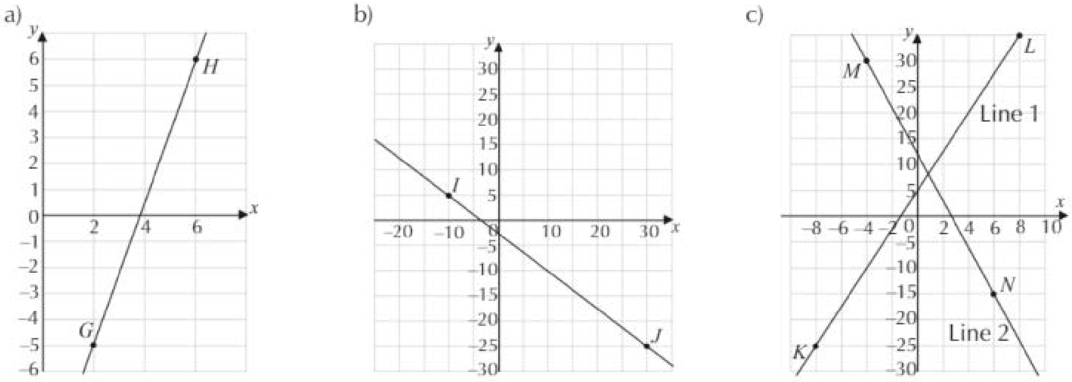
Gradients

Week 3 Task 3

HM: 201, 202, 203

Exercise 1

Q3 For each line shown below: (i) Use the axes to find the coordinates of each of the marked points.
(ii) Find the gradient of each of the lines.



Q4 a) Plot the points $U(-1, 2)$ and $V(2, 5)$ on a grid.

b) Find the gradient of the line joining points U and V.

Q5 a) Find the difference between the y-coordinates of the points $Y(2, 0)$ and $Z(-4, -3)$.

b) Find the difference between the x-coordinates of Y and Z.

c) Hence find the gradient of the line containing points Y and Z.

Gradients

Week 3 Task 3

HM: 201, 202, 203

Exercise 1

Q6 Find the gradients of the lines joining the following points.

a) $A(0, 4)$, $B(2, 10)$

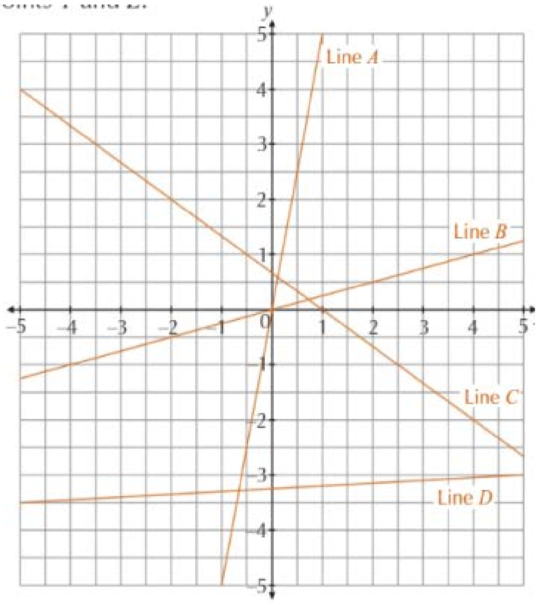
b) $C(1, 3)$, $D(5, 11)$

c) $E(-1, 3)$, $F(5, 7)$

d) $G(-3, -2)$, $H(1, -5)$

e) $I(5, -2)$, $J(1, 1)$

f) $K(-4, -3)$, $L(-8, -6)$



Q7 Find the gradients of lines A-D on the graph on the right.

Equations of Linear Graphs

Week 3 Task 4

HM: 207, 208, 209, 210

The equation of a straight line can be written in the form $y = mx + c$.
E.g. for $y = 3x + 5$, $m = 3$ and $c = 5$.

When written in this form:

- m is the **gradient** of the line,
- c tells you the **y-intercept** — the point where the line crosses the y-axis.

If you're given an equation that isn't in $y = mx + c$ form, rearrange it into this format so that you can read off the values of m and c. For example:

Equation	$y = mx + c$ form	m	c
$y = 2 + 3x$	$y = 3x + 2$	3	2
$x - y = 0$	$y = x + 0$	1	0
$4x - 3 = 5y$	$y = \frac{4}{5}x - \frac{3}{5}$	$\frac{4}{5}$	$-\frac{3}{5}$

Make sure you don't mix up m and c when you get something like $y = 5 + 2x$. Remember, m is the number in front of the x and c is the number on its own. Watch out for **minus signs** too — both m and c can be **negative** (e.g. $y = -2x - 5$), so you have to include the minus sign when you state the gradient and y-intercept.

Equations of Linear Graphs

Week 3 Task 4

HM: 207, 208, 209, 210

Example 1

Write down the gradient and the coordinates of the y-intercept of $y = 2x + 1$.

The equation is already in the form $y = mx + c$, so you just need to read the values for the gradient and y-intercept from the equation.

$$y = \underset{m}{\boxed{2}}x + \underset{c}{\boxed{1}}$$

gradient = **2**

y-intercept = **(0, 1)**

Tip: The question asks for the coordinates of the y-intercept, so don't forget the x-coordinate (which is 0).

Equations of Linear Graphs

Week 3 Task 4

HM: 207, 208, 209, 210

Example 2

Find the gradient and the coordinates of the y-intercept of $2x + 3y = 12$.

1. Rearrange the equation into the form $y = mx + c$.
2. Write down the values for the gradient and y-intercept. Notice that m is negative, so the line slopes downwards from left to right.

$$\begin{aligned} 2x + 3y &= 12 &&\quad -\,2x \\ 3y &= -2x + 12 &&\quad \div\,3 \\ y &= \underset{m}{\boxed{-\tfrac{2}{3}}}x + \underset{c}{\boxed{4}} \end{aligned}$$

gradient = $\mathbf{-\frac{2}{3}}$

y-intercept = **(0, 4)**

Equations of Linear Graphs

Week 3 Task 4

HM: 207, 208, 209, 210

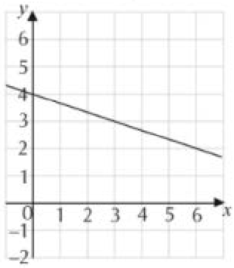
Exercise 1

Q1 Write down the gradient and the coordinates of the y-intercept for each of the following graphs.

a) $y = 2x - 4$
b) $y = 5x - 11$
c) $y = -3x + 7$
d) $y = 4x$
e) $y = \frac{1}{2}x - 1$
f) $y = -x - \frac{1}{2}$
g) $y = 3 - x$
h) $y = 3$

Q2 Match the graphs to the correct equation from the box.

$y = x + 2$
$y = \frac{7}{3}x - 1$
$y = -x + 6$
$y = 3x$
$y = -\frac{1}{3}x + 4$
$y = \frac{1}{3}x + 2$

A


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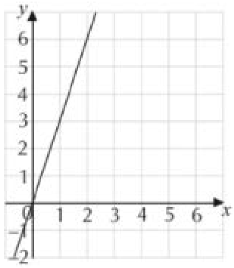

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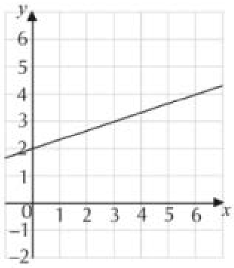

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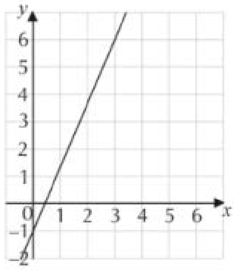

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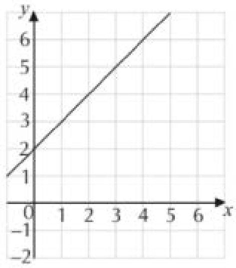

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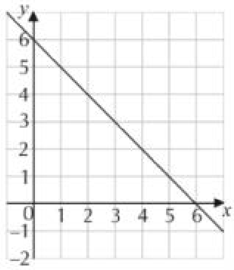


Equations of Linear Graphs

Week 3 Task 4

HM: 207, 208, 209, 210

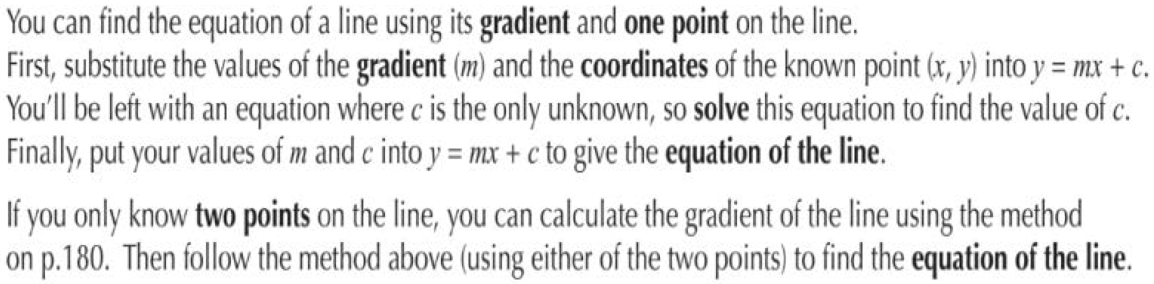
Exercise 1

Q3 Find the gradient and the coordinates of the y-intercept for each of the following graphs.

a) $3y = 9 - 3x$
b) $y - 5 = 7x$
c) $y + x = 8$
d) $x = 6 + 2y$
e) $3x + y = 1$
f) $3y - 6x = 15$
g) $4x = 5y - 5$
h) $8x - 2y = 14$
i) $5x + 4y = -3$
j) $4y - 6x + 8 = 0$
k) $6x - 3y + 1 = 0$
l) $\frac{1}{2} = -4x - 2y$

Equations of Linear Graphs

Week 3 Task 4



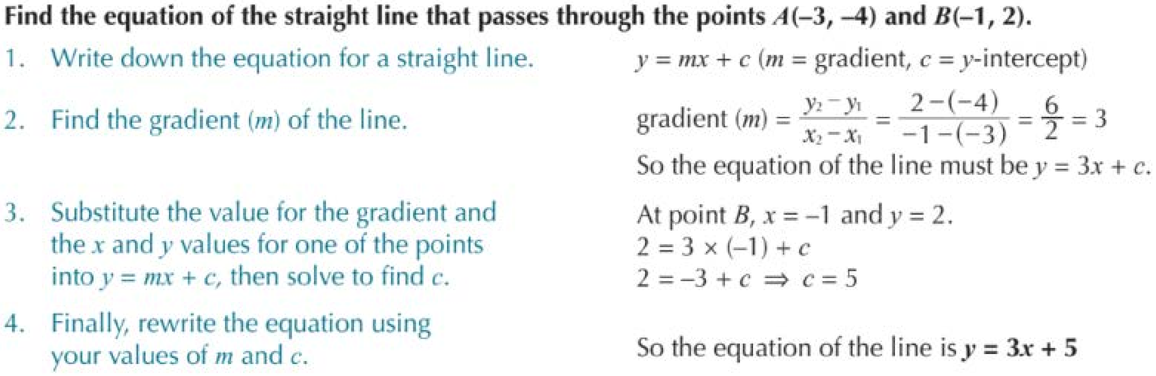
You can find the equation of a line using its **gradient** and **one point** on the line.
First, substitute the values of the **gradient** (m) and the **coordinates** of the known point (x, y) into $y = mx + c$.
You'll be left with an equation where c is the only unknown, so **solve** this equation to find the value of c.
Finally, put your values of m and c into $y = mx + c$ to give the **equation of the line**.

If you only know **two points** on the line, you can calculate the gradient of the line using the method on p.180. Then follow the method above (using either of the two points) to find the **equation of the line**.

Equations of Linear Graphs

Week 3 Task 4
HM: 211, 212, 213

Example 3



Find the equation of the straight line that passes through the points $A(-3, -4)$ and $B(-1, 2)$.

1. Write down the equation for a straight line. — $y = mx + c$ (m = gradient, c = y-intercept)
2. Find the gradient (m) of the line. — gradient (m) = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{-1-(-3)} = \frac{6}{2} = 3$
   So the equation of the line must be $y = 3x + c$.
3. Substitute the value for the gradient and the x and y values for one of the points into $y = mx + c$, then solve to find c. — At point B, $x = -1$ and $y = 2$.
   $2 = 3 \times (-1) + c$
   $2 = -3 + c \Rightarrow c = 5$
4. Finally, rewrite the equation using your values of m and c. — So the equation of the line is $\boldsymbol{y = 3x + 5}$

Equations of Linear Graphs

Week 3 Task 4

HM: 211, 212, 213

Exercise 2

Q1 Find the equations of the following lines based on the information given.

a) gradient = 8, passes through (0, 2)

b) gradient = –1, passes through (0, 7)

c) gradient = 3, passes through (1, 10)

d) gradient = $\frac{1}{2}$, passes through (4, –5)

e) gradient = –7, passes through (2, –4)

f) gradient = 5, passes through (–3, –7)

Q2 Find the equations of the lines passing through the following points.

a) (3, 7) and (5, 11)

b) (5, 1) and (2, –5)

c) (4, 1) and (–3, –6)

d) (–2, 1) and (1, 7)

e) (2, 8) and (–1, –1)

f) (–3, 2) and (–2, 5)

g) (–7, 8) and (–1, 2)

h) (2, –1) and (4, –9)

i) (3, 4) and (–5, –8)

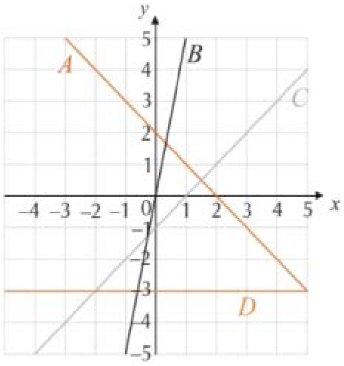
Equations of Linear Graphs

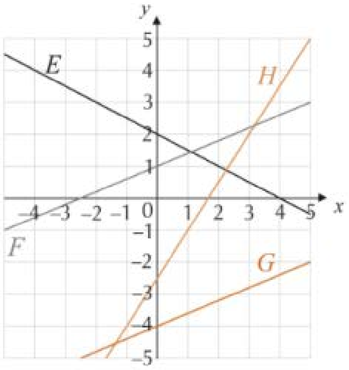
Week 3 Task 4

HM: 211, 212, 213

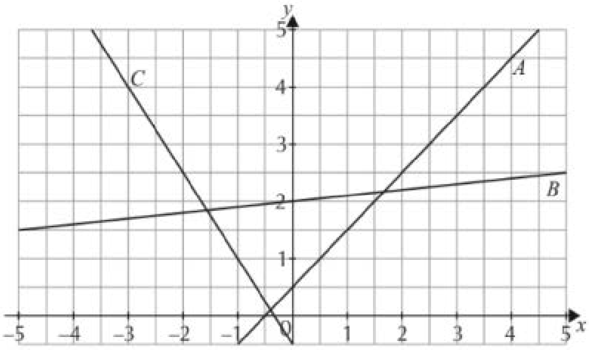
Exercise 2

Q3 Find the equations of the lines A to H shown below. Write all your answers in the form $y = mx + c$.





Q4 Find the equations for the lines shown on the graph on the right.
Give your answers in the form $y = mx + c$.



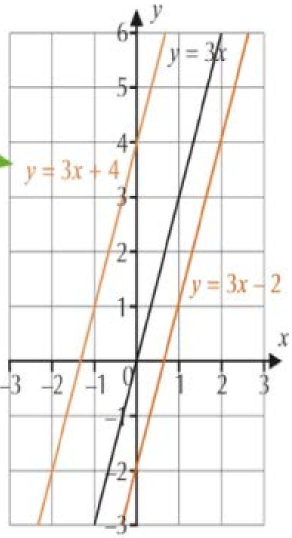
Parallel & Perpendicular Lines

Lines that are **parallel** have the **same gradient** — so their equations (in $y = mx + c$ form) all have the same value of $\boldsymbol{m}$.

For example, the lines $y = 3x$, $y = 3x - 2$ and $y = 3x + 4$ are all parallel.

To **check** if two lines are parallel, **rearrange** their equations so that they're both in $y = mx + c$ form, then **compare** the values of m.

If you have two parallel lines, A and B, and you know the **equation** of line A and **one point** on line B, you can find the equation of line B. First, find the **gradient** of line A (you can just read off this value if it's in $y = mx + c$ form). You know that line B has the **same gradient** as line A (as they're parallel), and you know the **coordinates** of one point on line B, so now you can use the method from p.183 to find the equation of line B.



Parallel & Perpendicular Lines

Example 1

Which of the following lines is parallel to the line $2x + y = 5$?

A: $y = 3 - 2x$ B: $x + y = 5$ C: $y - 2x = 6$

1. Rearrange the equation into the form $y = mx + c$ to find its gradient.

$2x + y = 5$
$y = 5 - 2x$
$y = -2x + 5$, so the gradient (m) = –2.

2. Rearrange the other equations in the same way. Any that have $m = -2$ will be parallel to $2x + y = 5$.

A: $y = 3 - 2x$
$y = -2x + 3$
$m = -2$

B: $x + y = 5$
$y = -x + 5$
$m = -1$

C: $y - 2x = 6$
$y = 2x + 6$
$m = 2$

So **line A** is parallel to $2x + y = 5$.

Parallel & Perpendicular Lines

Example 2

Find the equation of line L, which passes through the point (5, 8) and is parallel to $y = 3x + 2$.

1. Find the gradient of the line $y = 3x + 2$.
2. The lines are parallel, so line L will have the same gradient.
3. Substitute the values for x and y at the point (5, 8) into the equation. Solve to find c and hence the equation of line L.

Equation of a straight line: $y = mx + c$
The gradient of $y = 3x + 2$ is 3.

So the equation of line L must be $y = 3x + c$.

At (5, 8), $x = 5$ and $y = 8$:
$8 = 3(5) + c$
$8 = 15 + c$
$c = -7$, so the equation of line L is $\mathbf{y = 3x - 7}$.

Parallel & Perpendicular Lines

Exercise 1

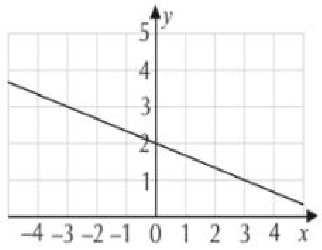
Q1 Write down the equations of three lines that are parallel to: a) $y = 5x - 1$ b) $x + y = 7$

Q2 Work out which of the following are the equations of lines parallel to: a) $y = 2x - 1$ b) $2x - 3y = 0$

A: $y - 2x = 4$
B: $2y = 2x + 5$
C: $2x - y = 2$
D: $2x + y + 7 = 0$
E: $3y + 2x = 2$
F: $6x - 9y = -2$

Q3 Which of the lines listed below are parallel to the line shown in the diagram?

A: $y + 3x = 2$
B: $3y = 7 - x$
C: $y = 4 - 3x$
D: $x - 3y = 8$
E: $y = 3 - \frac{1}{3}x$
F: $6y = -2x$



Q4 For each of the following, find the equation of the line which is parallel to the given line and passes through the given point. Give your answers in the form $y = mx + c$.

a) $y = 5x - 7$, (1, 8)
b) $y = 2x$, (−1, 5)
c) $y = \frac{1}{2}x + 3$, (6, −7)
d) $y = 8x - 1$, (−3, −5)
e) $2y = 6x + 3$, (−3, 4)
f) $y = 7 - 9x$, (1, −11)
g) $x + y = 4$, (8, 8)
h) $2x + y = 12$, (−4, 0)
i) $x + 3y + 1 = 0$, (−9, 9)

Parallel & Perpendicular Lines

Week 3 Task 5

HM: 214, 215, 216

Example 3

Find the equation of line B, which is perpendicular to $y = 5x - 2$ and passes through the point (10, 4).

1. Use the gradient of $y = 5x - 2$ to find the gradient of line B.	The line $y = 5x - 2$ has gradient $m = 5$. So the gradient of line B is $-\frac{1}{m} = -\frac{1}{5}$.
2. Substitute this gradient into the equation for a straight line along with the values for x and y at the point given.	The equation of line B is $y = -\frac{1}{5}x + c$. At the point (10, 4), $x = 10$ and $y = 4$ $4 = -\frac{1}{5}(10) + c$
3. Solve this equation to find c and hence the equation of line B.	$4 = -2 + c$ $c = 6$ So the equation of line B is $\mathbf{y = -\frac{1}{5}x + 6}$.

Parallel & Perpendicular Lines

Week 3 Task 5

HM: 214, 215, 216

Exercise 2

Q1 Find the gradient of a line which is perpendicular to a line with gradient:

a) 6
b) -3
c) $-\frac{1}{4}$
d) 12
e) -7
f) $\frac{2}{3}$
g) -2
h) 1.5
i) 0.3
j) -4.5
k) $-\frac{4}{3}$
l) $3\frac{1}{2}$

Q2 Write down the equation of any line which is perpendicular to:

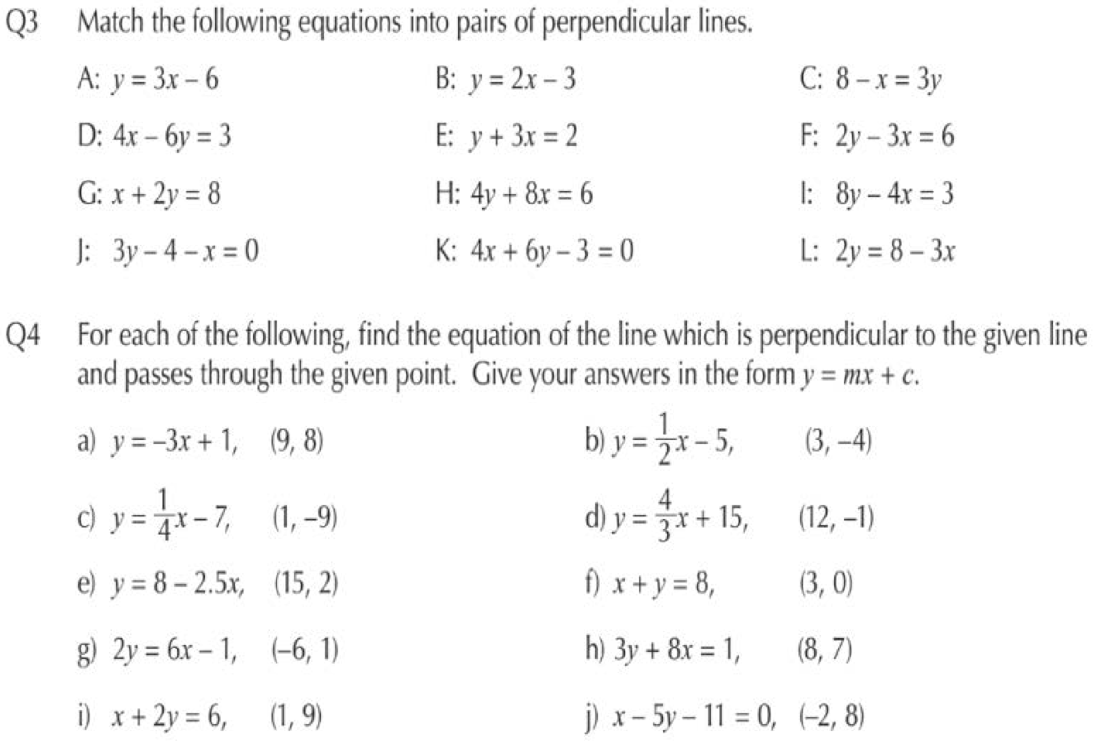
a) $y = 2x + 3$
b) $y = -3x + 11$
c) $y = 5 - 6x$
d) $2y = 5x + 1$
e) $x + y = 2$
f) $5x - 10y = 4$

Parallel & Perpendicular Lines

Week 3 Task 5

HM: 214, 215, 216

Exercise 2



Q3 Match the following equations into pairs of perpendicular lines.

A: $y = 3x - 6$

B: $y = 2x - 3$

C: $8 - x = 3y$

D: $4x - 6y = 3$

E: $y + 3x = 2$

F: $2y - 3x = 6$

G: $x + 2y = 8$

H: $4y + 8x = 6$

I: $8y - 4x = 3$

J: $3y - 4 - x = 0$

K: $4x + 6y - 3 = 0$

L: $2y = 8 - 3x$

Q4 For each of the following, find the equation of the line which is perpendicular to the given line and passes through the given point. Give your answers in the form $y = mx + c$.

a) $y = -3x + 1$, $(9, 8)$

b) $y = \frac{1}{2}x - 5$, $(3, -4)$

c) $y = \frac{1}{4}x - 7$, $(1, -9)$

d) $y = \frac{4}{3}x + 15$, $(12, -1)$

e) $y = 8 - 2.5x$, $(15, 2)$

f) $x + y = 8$, $(3, 0)$

g) $2y = 6x - 1$, $(-6, 1)$

h) $3y + 8x = 1$, $(8, 7)$

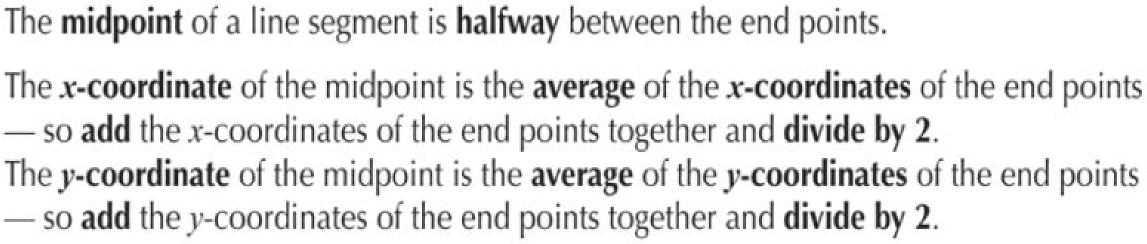
i) $x + 2y = 6$, $(1, 9)$

j) $x - 5y - 11 = 0$, $(-2, 8)$

Line Segments

Week 3 Task 6

HM: 200



The **midpoint** of a line segment is **halfway** between the end points.

The ***x*-coordinate** of the midpoint is the **average** of the ***x*-coordinates** of the end points — so **add** the *x*-coordinates of the end points together and **divide by 2**.
The ***y*-coordinate** of the midpoint is the **average** of the ***y*-coordinates** of the end points — so **add** the *y*-coordinates of the end points together and **divide by 2**.

Line Segments

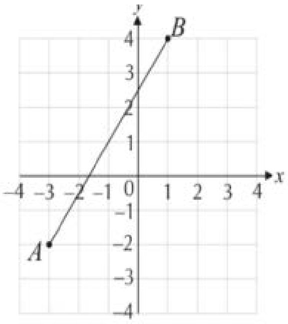
Week 3 Task 6

HM: 200

Example 1

Find the midpoint of the line segment AB, shown on the right.

1. Write down the coordinates of the end points A and B.	$A(-3, -2)$ and $B(1, 4)$
2. Find the average of the x-coordinates by adding them together and dividing by 2. Find the average of the y-coordinates in the same way.	$\left(\frac{-3+1}{2}, \frac{-2+4}{2}\right) = (-1, 1)$ The midpoint has coordinates **(–1, 1)**.



Line Segments

Week 3 Task 6

HM: 200

Exercise 1

Q1 Find the coordinates of the midpoint of the line segment AB, where A and B have coordinates:

a) $A(8, 0)$, $B(4, 6)$

b) $A(-2, 3)$, $B(6, 5)$

c) $A(4, -7)$, $B(-2, 1)$

d) $A(-3, 0)$, $B(9, -2)$

e) $A(-6, -2)$, $B(-4, 6)$

f) $A(-1, 3)$, $B(-1, -7)$

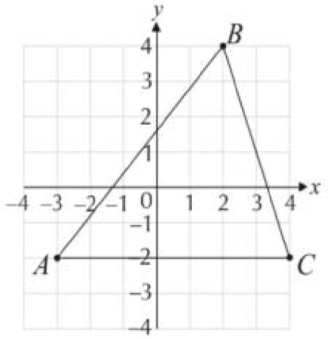
g) $A(-\frac{1}{2}, 4)$, $B(\frac{1}{2}, -3)$

h) $A(2p, q)$, $B(6p, 7q)$

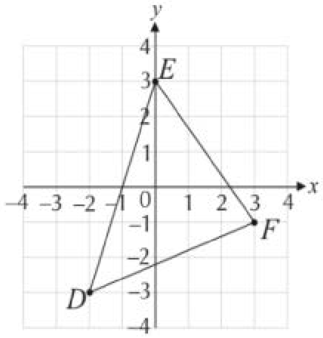
i) $A(8p, 2q)$, $B(2p, 14q)$

Q2 Find the midpoint of each side of the following triangles.

a)



b)



Line Segments

Week 3 Task 6

HM: 200

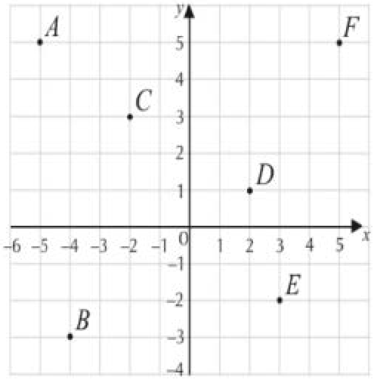
Exercise 1

Q3 Point A has the coordinates $A(1, 8)$. The midpoint, M, of the line segment AB has the coordinates $M(5, 3)$. Find the coordinates of B.

Q4 The coordinates of the endpoint, C, and midpoint, M, of the line segment CD are $C(6, -7)$ and $M(2, -1)$. Find the coordinates of point D.

Q5 Use the diagram on the right to find the midpoints of the following line segments.

a) AF
b) AC
c) DF
d) BE
e) BF
f) CE



Line Segments

Week 3 Task 6

HM: n/a

To find the **length** of a line segment (or the **distance** between two points), use **Pythagoras' theorem**:

1. Create a **right-angled triangle** using the line segment as the **hypotenuse**.
2. Work out the lengths of the two **shorter sides** — find the **difference** between the ***x*-coordinates** of the end points to find the length of the **horizontal** side, then find the **difference** between the ***y*-coordinates** of the end points to find the length of the **vertical** side.
3. Put these values into Pythagoras' theorem as a and b: $a^2 + b^2 = h^2$ and solve to find h.

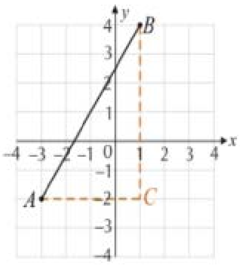
Line Segments

Week 3 Task 6

HM: n/a

Example 2

Calculate the length of the line segment *AB*. Give your answer to three significant figures.



1. Think of the line segment as the hypotenuse of a right-angled triangle *ABC*.
2. Calculate the length of *AC* and *BC*.
3. Calculate the length of *AB* using Pythagoras' theorem.

Length of $AC = 1 - (-3) = 4$

Length of $BC = 4 - (-2) = 6$

$AB = \sqrt{AC^2 + BC^2}$

$AB = \sqrt{4^2 + 6^2}$

$AB = \sqrt{52}$ = **7.21** (to 3 s.f.)

Line Segments

Week 3 Task 6

HM: n/a

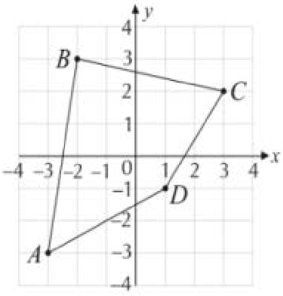
Exercise 2

Q1 Find the length of the line segments with the following end point coordinates. Give your answers to 3 significant figures.

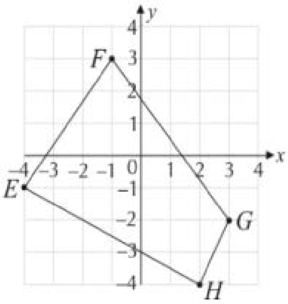
a) (5, 9) and (1, 6)
b) (15, 3) and (11, 8)
c) (5, 4) and (4, 1)
d) (3, 7) and (3, 14)
e) (–1, 9) and (9, –3)
f) (9, –4) and (–1, 12)
g) (1, –2) and (8, 2)
h) (–3, 7) and (–2, –3)
i) (–1, –1), (–5, 9)
j) (2, 4) and (–1, –4)
k) (0, –1) and (4, 8)
l) (–2, –1) and (11, 8)

Q2 Find the length of each side of the shapes below. Give your answers to 3 significant figures.

a)



b)



Line Segments

Week 3 Task 6
HM: n/a

Ratios can be used to express where a **point** is on a **line**. For example, if point R is a point on the line PQ such that $PR : RQ = 2 : 3$, then R is $\frac{2}{2+3} = \frac{2}{5}$ of the way from P to Q.

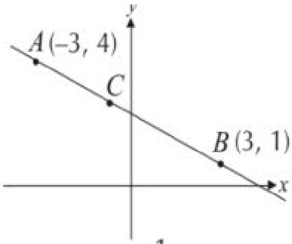
You can use ratios to find the **coordinates** of a point by treating the x- and y-coordinates **separately**.

Line Segments

Week 3 Task 6
HM: n/a

Example 3

Points A, B and C lie on a straight line. Point A has coordinates $(-3, 4)$ and point B has coordinates $(3, 1)$. Point C lies on line segment AB such that $AC : CB = 1:2$. Find the coordinates of point C.



1. Find the difference between the x- and y-coordinates of A and B.	x difference: $3 - (-3) = 6$ y difference: $1 - 4 = -3$
2. Use the given ratio to see how far along AB point C is.	C is $\frac{1}{1+2} = \frac{1}{3}$ of the way along AB, so x: $\frac{1}{3} \times 6 = 2$ y: $\frac{1}{3} \times -3 = -1$
3. Add the results to the coordinates of A to get C.	x-coordinate: $-3 + 2 = -1$ y-coordinate: $4 + -1 = 3$ Point C lies at **$(-1, 3)$**.

Line Segments

Week 3 Task 6

HM: n/a

Exercise 3

Q1 Point C lies on the line segment AB. Find the coordinates of C given that:

a) $A(3, -3)$, $B(6, 6)$ $AC:CB = 1:2$

b) $A(-3, 5)$, $B(9, 1)$ $AC:CB = 3:1$

c) $A(0, 4)$, $B(10, -1)$ $AC:CB = 2:3$

d) $A(-20, 1)$, $B(8, -13)$ $AC:CB = 3:4$

Q2 Each set of three points below lies on a straight line. Use the points to find the specified ratios.

a) Find $AB:BC$ given $A(0, 0)$, $B(2, 2)$ and $C(6, 6)$.

b) Find $DE:EF$ given $D(1, 0)$, $E(-3, 4)$ and $F(-4, 5)$.

c) Find $GH:HI$ given $G(-1, -2)$, $H(5, 2)$ and $I(14, 8)$.

Q3 Point T lies on the line segment SU. Find the coordinates of U given that:

a) $S(6, 2)$, $T(12, -4)$ $ST:TU = 3:2$

b) $S(-2, -4)$, $T(18, 11)$ $ST:TU = 5:4$

Quadratic Graphs

Week 3 Task 7

HM: 251, 252, 253, 254

Quadratic functions always have x^2 as the highest power of x. The **graphs** of quadratic functions are always the same shape (called a **parabola**) and are **symmetrical** about their lowest (or highest) point.

- If the coefficient of x^2 is **positive** (e.g. $y = 2x^2 - 3x - 1$), the parabola is **u-shaped**.
- If the coefficient of x^2 is **negative** (e.g. $y = -2x^2 + 3x + 1$), the parabola is **n-shaped**.

To **draw the graph** of a quadratic function $y = ax^2 + bx + c$, calculate y-values for a set of x-values in a **table**, plot the pairs of values as **coordinates** on a set of axes, then draw a **smooth curve** through the points. You can then **read off** the graph to estimate the value of x for a given value of y.

Quadratic Graphs

Week 3 Task 7

HM: 251, 252, 253, 254

Example 1

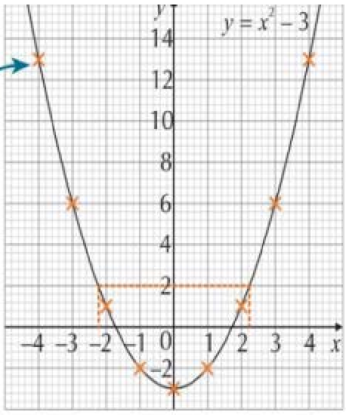
Draw the graph of $y = x^2 - 3$ and use it to estimate x when $y = 2$.

1. Use a table to find the coordinates of points on the graph.

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$x^2 - 3$	13	6	1	–2	–3	–2	1	6	13

So the coordinates are (–4, 13), (–3, 6), etc.

2. Plot the coordinates on a suitable set of axes.
Join the points with a smooth curve (not straight lines).



3. Draw a horizontal line at $y = 2$ and read off the values of x: **$x = \pm 2.2$** (1 d.p.)

Quadratic Graphs

Week 3 Task 7

HM: 251, 252, 253, 254

Exercise 1

Q1 For the following quadratic equations, copy and complete the table and draw each graph.

a) $y = 6 - x^2$

x	–4	–3	–2	–1	0	1	2	3	4
x^2			4			1		9	
$6 - x^2$			2			5		–3	

b) $y = 2x^2$

x	–4	–3	–2	–1	0	1	2	3	4
$2x^2$		18				2			

Q2 Draw the graph of $y = x^2 + 5$ for values of x between –4 and 4.

a) Use your graph to find the value of y when: (i) $x = 2.5$ (ii) $x = -0.5$

b) Use your graph to find the values of x when: (i) $y = 6.5$ (ii) $y = 10$

Q3 Draw the graph of $y = 4 - x^2$ for values of x between –4 and 4.
Write down the values of x where the graph crosses the x-axis.

Q4 Draw the graph of $y = 3x^2 - 11$ for values of x between –4 and 4.
Estimate the values of x where the graph crosses the x-axis.

Quadratic Graphs

Week 3 Task 7

HM: 251, 252, 253, 254

Example 2

Draw the graph of $y = x^2 + 3x - 2$.

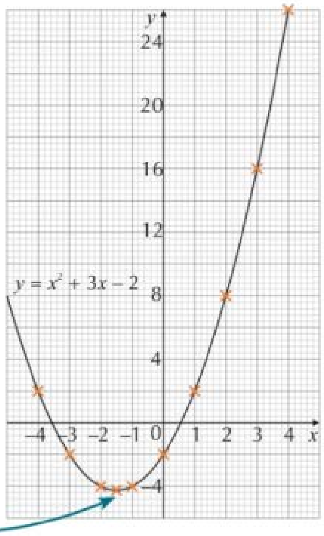
1. Add extra rows to the table to make it easier to work out the y-values.

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$+3x$	–12	–9	–6	–3	0	3	6	9	12
–2	–2	–2	–2	–2	–2	–2	–2	–2	–2
$x^2 + 3x - 2$	2	–2	–4	–4	–2	2	8	16	26

2. The table doesn't tell you the lowest point on the curve, so you need to find one more point before you can draw the graph. Quadratic graphs are always symmetrical, so the x-coordinate of the lowest point on the curve is halfway between the two lowest points from the table (or any pair of points with the same y-coordinate).

   So the lowest point of the graph is halfway between $x = -2$ and $x = -1$, when $x = -1.5$ and $y = (-1.5)^2 + (3 \times -1.5) - 2 = -4.25$.

3. Plot the points and join with a smooth curve.



Quadratic Graphs

Week 3 Task 7

HM: 251, 252, 253, 254

Exercise 2

Q1 Copy and complete the table and draw the graph of $y = 2x^2 + 3x - 7$.

x	–4	–3	–2	–1	0	1	2	3	4
$2x^2$		18						18	
$+3x$		–9						9	
–7		–7						–7	
$2x^2 + 3x - 7$		2						20	

Q2 Draw the graph of $y = x^2 - 5x + 3$ for values of x between –3 and 6.

a) Use your graph to find the value of y when: (i) $x = -1.5$ (ii) $x = 1.5$

b) Use your graph to find the values of x when: (i) $y = 8$ (ii) $y = -2$

Q3 Draw the graph of $y = 11 - 2x^2$ for values of x between –4 and 4.

a) Use your graph to find the value of y when: (i) $x = -2.5$ (ii) $x = 1.25$

b) Use your graph to find the values of x when: (i) $y = 0$ (ii) $y = 11$

Quadratic Graphs

Week 3 Task 7

HM: 255, 256, 257

Quadratic graphs are always the same shape, which means you can **sketch** them without a table of coordinates. Draw a u-shaped or n-shaped **parabola** in roughly the correct position and label these **important points**:

- The **y-intercept** is the value of the **constant term** in the equation, i.e. $(0, c)$ for $y = ax^2 + bx + c$, because this is the value of y when $x = 0$.
- The **x-intercepts** can be found by **factorising** the equation. A quadratic $y = (x + p)(x + q)$ crosses the x-axis at $(-p, 0)$ and $(-q, 0)$, because $x = -p$ and $x = -q$ are the solutions to $(x + p)(x + q) = 0$.
- The **turning point** is the **lowest point** (for a u-shaped graph) or **highest point** (for an n-shaped graph) on the curve. Due to the symmetry of the graph, the **x-coordinate** of the turning point is always **halfway** between the **x-intercepts**. Put this value into the equation to find the y-coordinate.

Tip: The turning point lies on the vertical line of symmetry.

Quadratic Graphs

Week 3 Task 7

HM: 255, 256, 257

Example 3

Sketch the graph of $y = x^2 + x - 6$, and label the turning point and intercepts with their coordinates.

1. Find the y-intercept by putting $x = 0$ in the equation (or just look at the constant term).

   When $x = 0$, $y = 0^2 + 0 - 6 = -6$, so the y-intercept is at $(0, -6)$.

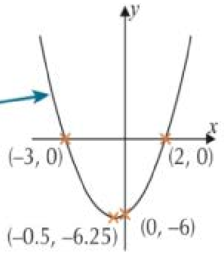
2. Factorise and solve the equation $x^2 + x - 6 = 0$ to find the x-intercepts.

   $x^2 + x - 6 = 0 \Rightarrow (x + 3)(x - 2) = 0 \Rightarrow x = -3$ and $x = 2$, so the x-intercepts are $(-3, 0)$ and $(2, 0)$.

3. Use symmetry to find the turning point. The x-coordinate is halfway between the x-intercepts. Find the y-coordinate by putting the x-coordinate into the equation.

   x-coordinate $= (2 + -3) \div 2 = -0.5$
   y-coordinate $= (-0.5)^2 + (-0.5) - 6 = -6.25$
   So the turning point has coordinates $(-0.5, -6.25)$.

4. Use all this information to sketch and label the graph. The x^2 term is positive so the graph is u-shaped. This means that the turning point will be the lowest point on the graph.



Tip: Sketches don't have to be completely accurate — they just need the correct general shape, and key points labelled and in roughly the correct positions.

Quadratic Graphs

Week 3 Task 7

HM: 255, 256, 257

Exercise 3

Q1 For each of the following equations, find the coordinates of: (i) the intercepts, (ii) the turning point.

a) $y = (x - 1)(x + 1)$ b) $y = (x + 7)(x + 1)$ c) $y = x^2 + 16x + 60$

Q2 Sketch the graphs of these equations. Label the turning points and intercepts with their coordinates.

a) $y = x^2 - 4$ b) $y = x^2 - 4x - 12$ c) $y = x^2 + 12x + 32$

d) $y = x^2 + x - 20$ e) $y = -x^2 - 2x + 3$ f) $y = -x^2 - 14x - 49$

g) $y = 2x^2 + 4x - 16$ h) $y = 5x^2 - 6x - 8$ i) $y = -2x^2 - x + 6$

Quadratic Graphs

Week 3 Task 7

HM: 255, 256, 257

You can use the **completed square** form of a quadratic equation to easily find the coordinates of the **turning point** of its graph. For equations of the form $y = (x + p)^2 + q$, the turning point can be found where the expression in **brackets** is **zero**, i.e. where $\boldsymbol{x = -p}$ and $\boldsymbol{y = q}$, at $\boldsymbol{(-p, q)}$.

This also tells you the **number of roots**. For a **positive** quadratic, if the minimum point lies **above** the x-axis (i.e. has a **positive** value of q), the graph won't cross the x-axis and so there are **no real roots**. (The same is true for a **negative** quadratic when the maximum point lies **below** the x-axis with a **negative** value of q.) If the turning point lies **on the x-axis** there is **one repeated root**. Otherwise there are **two real roots**.

Quadratic Graphs

Week 3 Task 7

HM: 255, 256, 257

Example 4

Find the coordinates of the turning point of the graph with equation $y = 4 + 2x - x^2$.

1. Complete the square.
   $y = 4 + 2x - x^2 = -(x^2 - 2x - 4)$
   $= -[(x - 1)^2 - 1 - 4] = 5 - (x - 1)^2$
2. Find the x- and y-values when the expression in brackets is zero.
   $(x - 1) = 0$, so $x = 1$, $y = 5 - 0 = 5$.
   So the turning point is at **(1, 5)**.

Tip: The x^2 term is negative, so the turning point is the highest point on the n-shaped graph.

Quadratic Graphs

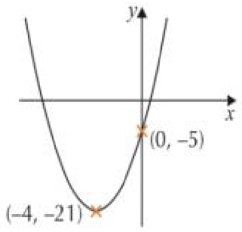
Week 3 Task 7

HM: 255, 256, 257

Example 5

Sketch the graph of $y = x^2 + 8x - 5$. Label the y-intercept and turning point.

1. Complete the square.
   $y = (x + 4)^2 - 16 - 5 = (x + 4)^2 - 21$
2. The turning point occurs when the brackets of the completed square are equal to 0.
   When $(x + 4) = 0$, $x = -4$, $y = 0 - 21 = -21$.
   So the turning point is at $(-4, -21)$.
3. Substitute $x = 0$ into $y = x^2 + 8x - 5$ to find the y-intercept.
   $y = 0^2 + (8 \times 0) - 5 = -5$, so the y-intercept is $(0, -5)$.
4. Sketch the graph through these two points and label their coordinates.
   The x^2 term is positive, so the graph is u-shaped, and the turning point is the lowest (minimum) point on the graph.



Tip: Here, the turning point lies below the x-axis, so the graph crosses the axis twice and there are 2 real roots, at $(x + 4)^2 - 21 = 0$ $\Rightarrow x = -4 \pm \sqrt{21}$.

Quadratic Graphs

Week 3 Task 7
HM: 255, 256, 257

Exercise 4

Q1 For each of the following equations, find the coordinates of: (i) the y-intercept, (ii) the turning point.

a) $y = (x + 5)^2 - 9$
b) $y = (x - 3)^2 - 30$
c) $y = (x - 4)^2 - 13$

Q2 By completing the square, sketch the graphs of the following equations.
Label the y-intercept and turning point of each graph.

a) $y = x^2 - 6x - 5$
b) $y = x^2 - 4x + 2$
c) $y = x^2 + 8x - 6$
d) $y = x^2 + 2x + 8$
e) $y = x^2 - x + 10$
f) $y = -x^2 + 10x - 6$

Week 3 Assessment

Question 1

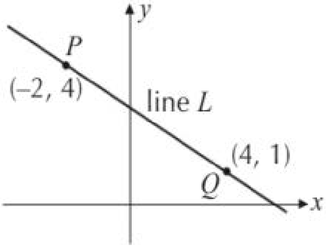
Line A has equation $5x + 2y - 8 = 0$.
Line B is parallel to line A but has a y-intercept which is triple that of line A.
Find the equation of line B.

[3 marks]

Week 3 Assessment

Question 2

Line L passes through the points P and Q, as shown below.



a) Find the equation of line L.

[3 marks]

b) Lines L and M are perpendicular and intersect at the point (2, 2). Find the equation of line M.

[3 marks]

Week 3 Assessment

Question 3

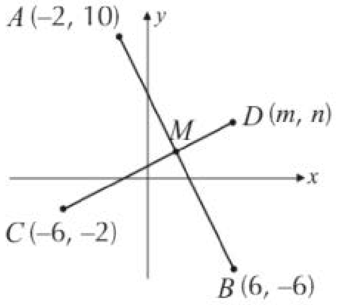
A straight line passes through the points (9, 110) and (–5, –100).
Does the point (33, 450) lie on the line? Justify your answer.

[4 marks]

Week 3 Assessment

Question 4

The diagram below shows lines AB and CD.
Line CD intersects AB at the midpoint, M, of line AB.



a) Find the midpoint, M, of line AB.

[2 marks]

b) Given that $CM : MD = 2 : 1$, find the length of the line segment CD. Give your answer to one decimal place.

[3 marks]

Week 3 Assessment

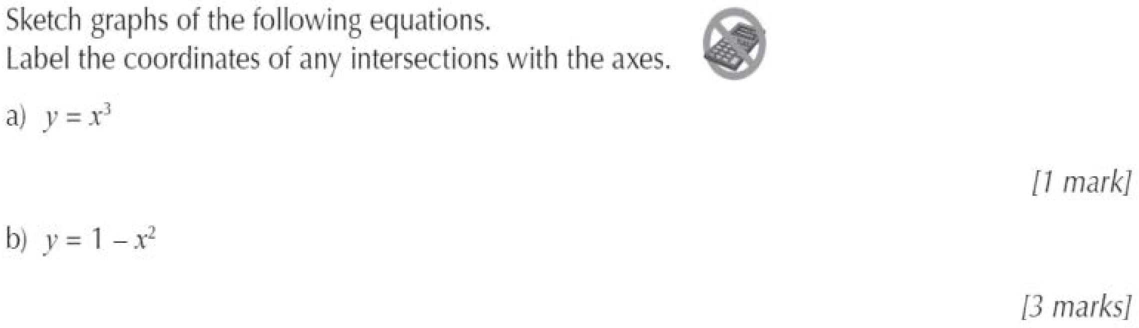
Question 5

A triangle has vertices $A(1, 5)$, $B(3, -1)$ and $C(6, 0)$.
Is triangle ABC right-angled? Justify your answer.

[4 marks]

Week 3 Assessment

Question 6



Sketch graphs of the following equations.
Label the coordinates of any intersections with the axes.

a) $y = x^3$

[1 mark]

b) $y = 1 - x^2$

[3 marks]

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Year 11 Transition Tasks to A Level Mathematics

Week 4 Tasks

1. Pythagoras' Theorem HM: 498, 499, 500, 501, 502, 503, 504

2. Pythagoras' Theorem in 3D HM: 505, 506, 507

3. Trigonometry - Sin, Cos and Tan HM: 508, 509,510, 511, 512, 513, 514, 515

4. The Sine and Cosine Rules HM: 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530

5. Trigonometry in 3D HM: 856, 857, 858, 859, 860

Pythagoras' Theorem

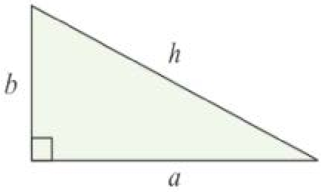
Week 4 Task 1

HM: 498, 499, 500, 501

The lengths of the sides in a **right-angled triangle** always follow the rule: $h^2 = a^2 + b^2$

h is the **hypotenuse** — this is the **longest** side, which is always **opposite** the right angle. a and b are the **shorter** sides.

This is **Pythagoras' theorem** and it is used to find lengths in right-angled triangles.



To find the length of the hypotenuse in a right-angled triangle:

- **Square** the lengths of sides a and b.
- **Add** together the squared lengths, a^2 and b^2, to get h^2.
- Take the **square root** of h^2 to find the hypotenuse, h.

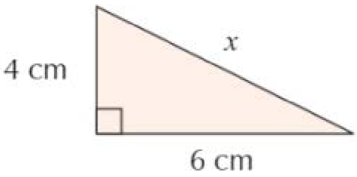
Pythagoras' Theorem

Week 4 Task 1

HM: 498, 499, 500, 501

Example 1

Find the exact length of x on the triangle shown.



1. Substitute the values from the diagram into the formula.

$$h^2 = a^2 + b^2$$
$$x^2 = 6^2 + 4^2$$

2. Add together the squared lengths to get x^2.

$$x^2 = 36 + 16 = 52$$

3. Work out the square root to find x.

$$x = \sqrt{52} = \sqrt{4 \times 13}$$
$$= \mathbf{2\sqrt{13}\ cm}$$

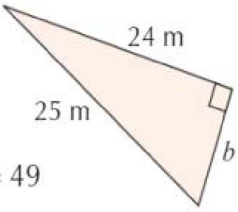
Pythagoras' Theorem

Week 4 Task 1

HM: 498, 499, 500, 501

Example 2

Find the length of b on the triangle shown on the right.



1. Substitute the values from the diagram into the formula. $h^2 = a^2 + b^2$, $25^2 = 24^2 + b^2$
2. Rearrange to make b^2 the subject. $b^2 = 25^2 - 24^2 = 625 - 576 = 49$
3. Work out the square root to find b. $b = \sqrt{49} = \mathbf{7\ m}$

Pythagoras' Theorem

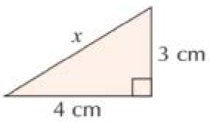
Week 4 Task 1

HM: 498, 499, 500, 501

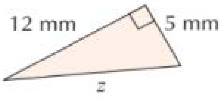
Exercise 1

Q1 Find the exact length of the hypotenuse in each of the triangles below.

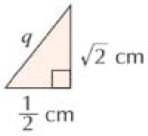
a)



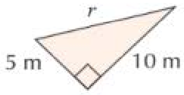
b)



c)

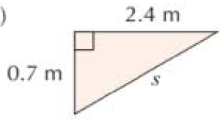


d)

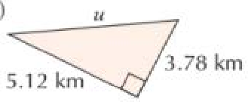


Q2 Find the length of the hypotenuse in each of the triangles below.
Give your answers correct to 2 decimal places where appropriate.

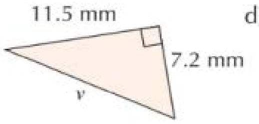
a)



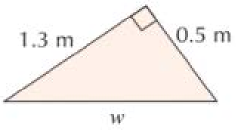
b)



c)

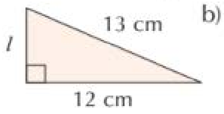


d)

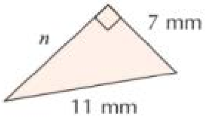


Q3 Find the exact lengths of the unknown sides in these triangles.

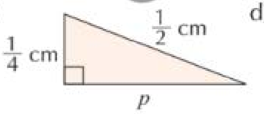
a)



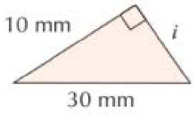
b)



c)



d)

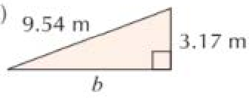


Q4 Find the lengths of the unknown sides in these triangles. Give your answers correct to 2 d.p.

a)

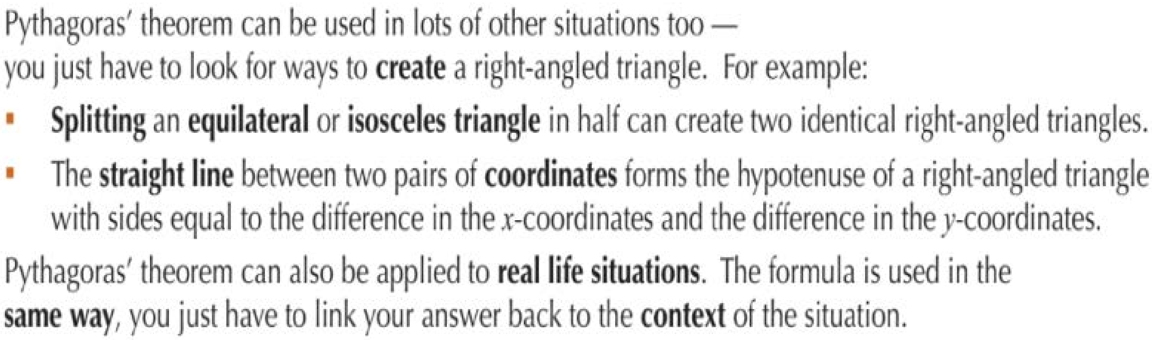
b)

c)



d)

Pythagoras' Theorem



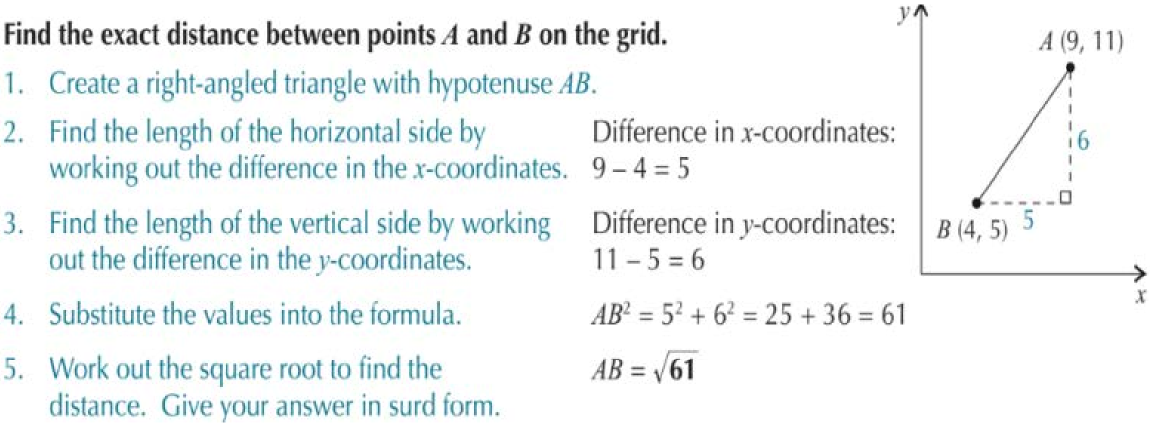
Pythagoras' theorem can be used in lots of other situations too — you just have to look for ways to **create** a right-angled triangle. For example:

- **Splitting** an **equilateral** or **isosceles triangle** in half can create two identical right-angled triangles.
- The **straight line** between two pairs of **coordinates** forms the hypotenuse of a right-angled triangle with sides equal to the difference in the x-coordinates and the difference in the y-coordinates.

Pythagoras' theorem can also be applied to **real life situations**. The formula is used in the **same way**, you just have to link your answer back to the **context** of the situation.

Pythagoras' Theorem

Example 3



Find the exact distance between points A and B on the grid.

1. Create a right-angled triangle with hypotenuse AB.
2. Find the length of the horizontal side by working out the difference in the x-coordinates. Difference in x-coordinates: $9 - 4 = 5$
3. Find the length of the vertical side by working out the difference in the y-coordinates. Difference in y-coordinates: $11 - 5 = 6$
4. Substitute the values into the formula. $AB^2 = 5^2 + 6^2 = 25 + 36 = 61$
5. Work out the square root to find the distance. Give your answer in surd form. $AB = \sqrt{61}$

Pythagoras' Theorem

Week 4 Task 1

HM: 502, 503, 504

Example 4

A TV has a height of 40 cm and width of w cm.
Its diagonal measures 82 cm. Will the TV fit in a box 75 cm wide?

Tip: Sketch a diagram to help you understand the question, if needed.

1. The diagonal and sides form a right-angled triangle, so substitute the values into the formula. $h^2 = a^2 + b^2$ $82^2 = 40^2 + w^2$
2. Rearrange to make w^2 the subject. $w^2 = 82^2 - 40^2 = 6724 - 1600$ $= 5124$
3. Work out the square root to find w. $w = \sqrt{5124} = 71.58$ cm (2 d.p.)
4. Use your answer to draw a conclusion. $71.58 < 75$ so **yes, the TV will fit in the box.**

Pythagoras' Theorem

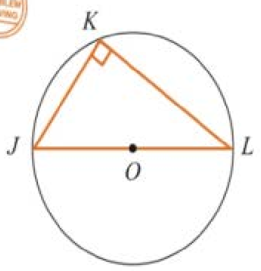
Week 4 Task 1

HM: 502, 503, 504

Exercise 2

Unless told otherwise, give your answers correct to 2 decimal places where appropriate.

Q1 An equilateral triangle has sides of length 10 cm. Find the perpendicular distance from a vertex to its opposite side.



Q2 The triangle JKL is drawn inside a circle centred on O, as shown. JK and KL have lengths 4.9 cm and 6.8 cm respectively.

a) Find the length of JL. b) Find the radius of the circle.

Q3 A kite gets stuck at the top of a vertical tree. The kite's 15 m string is taut, and its other end is held on the ground, 8.5 m from the base of the tree. Find the height of the tree.

Q4 Newtown is 88 km northwest of Oldtown. Bigton is 142 km from Newtown, and lies northeast of Oldtown. What is the distance from Bigton to Oldtown, to the nearest kilometre?

Q5 A triangle has sides measuring 1.5 m, 2 m and 2.5 m. Show that this is a right-angled triangle.

Pythagoras' Theorem

Week 4 Task 1

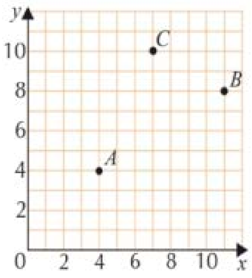
HM: 502, 503, 504

Exercise 2

Q6 A boat is rowed 200 m east and then 150 m south. If it had been rowed to the same point in a straight line instead, how much shorter would the journey have been?

Q7 The points *A*, *B* and *C* are shown on the right. Find the exact lengths of the line segments between the following pairs of points.

a) *A* and *B*　　b) *B* and *C*　　c) *A* and *C*

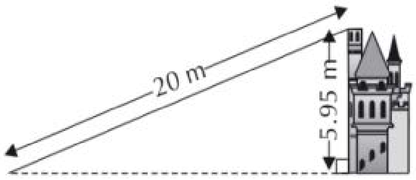


Q8 Find the exact distance between points *P* and *Q* with coordinates (11, 1) and (17, 19) respectively.

Q9 Kevin wants to set up a 20 m slide from the top of his 5.95 metre-high tower.

a) How far from the base of the tower should Kevin anchor the slide?

b) A safety inspector shortens the slide and anchors it 1.5 m closer to the tower. What is the new length of the slide?

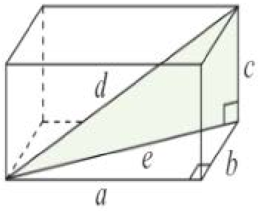


Pythagoras' Theorem in 3D

Week 4 Task 2

HM: 505, 506, 507

For a **cuboid** of length *a*, width *b* and height *c*, the length of the **longest diagonal**, *d*, can be found using the formula: $d^2 = a^2 + b^2 + c^2$

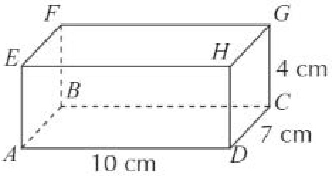


This formula comes from using the 2D Pythagoras' theorem **twice**.
In the diagram to the right, *a*, *b* and *e* make a **right-angled triangle** (*e* is the **diagonal** of one of the cuboid's faces and forms the **hypotenuse**), so $e^2 = a^2 + b^2$.
Side *c* makes a right-angled triangle with *d* and *e*, so $d^2 = e^2 + c^2 = a^2 + b^2 + c^2$.

Pythagoras' Theorem in 3D

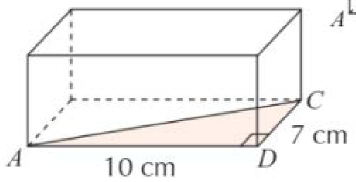
Example 1

Find the exact length *AG* in the cuboid shown on the right.



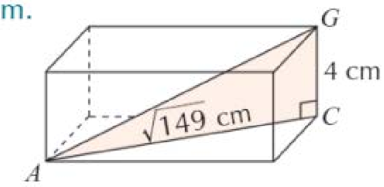
1. Use Pythagoras on the triangle *ACD* to find the length *AC*.

$AC^2 = 10^2 + 7^2 = 149$
$AC = \sqrt{149}$ cm



2. Now use Pythagoras again on the triangle *AGC* to find the length *AG*. Give your answer in surd form.

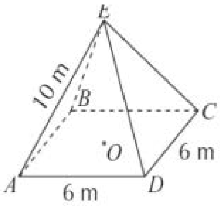
$AG^2 = (\sqrt{149})^2 + 4^2$
$= 149 + 16 = 165$
$AG = \sqrt{\mathbf{165}}$ **cm**



Pythagoras' Theorem in 3D

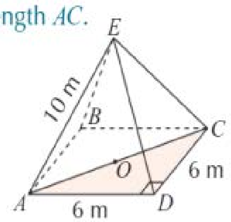
Example 2

A square-based pyramid *ABCDE* is shown on the right. Point *O* is the midpoint of the base *ABCD* and point *E* lies directly above *O*. Find the exact length *OE*.



1. Use Pythagoras on the triangle *ACD* to find the length *AC*.

$AC^2 = 6^2 + 6^2 = 72$
$AC = \sqrt{72} = 6\sqrt{2}$ m

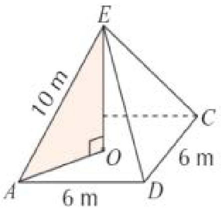


2. *O* is the midpoint of *AC*, so halve *AC* to get *AO*.

$AO = 6\sqrt{2} \div 2 = 3\sqrt{2}$ m

3. Use Pythagoras on the triangle *AOE* to find the length *OE*. Give your answer in surd form.

$AE^2 = AO^2 + OE^2$
$OE^2 = 10^2 - (3\sqrt{2})^2$
$= 100 - 18 = 82$
$OE = \sqrt{\mathbf{82}}$ **m**



Tip: You have to do this example in stages — you're not finding the diagonal of a cuboid, so you can't use the formula.

Pythagoras' Theorem in 3D

Week 4 Task 2

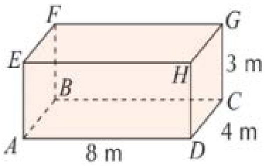
HM: 505, 506, 507

Exercise 1

Q1 The cuboid *ABCDEFGH* is shown on the right.

a) By considering the triangle *ABD*, find the exact length *BD*.

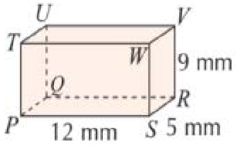
b) By considering the triangle *BFD*, find the exact length *FD*.



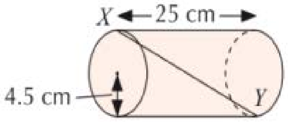
Q2 The cuboid *PQRSTUVW* is shown on the right.

a) Find the exact length *PR*.

b) Find the exact length *RT*.



Q3 A cylinder of length 25 cm and radius 4.5 cm is shown on the right. *X* and *Y* are points on opposite edges of the cylinder, such that *XY* is as long as possible. Find the length *XY* to 3 significant figures.



Q4 A cuboid measures 2.5 m × 3.8 m × 9.4 m. Find the length of the diagonal of the cuboid to 2 d.p.

Pythagoras' Theorem in 3D

Week 4 Task 2

HM: 505, 506, 507

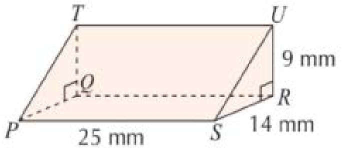
Exercise 1

In Questions 5-11, give all answers to 3 significant figures.

Q5 The triangular prism *PQRSTU* is shown on the right.

a) Find the length *QS*.

b) Find the length *ST*.



Q6 Find the length of the diagonal of a cube of side 5 m.

Q7 A pencil case in the shape of a cuboid is 16.5 cm long, 4.8 cm wide and 2 cm deep. What is the length of the longest pencil that will fit in the case? Ignore the thickness of the pencil.

Q8 A spaghetti jar is in the shape of a cylinder. The jar has radius 6 cm and height 28 cm. What is the length of the longest stick of dried spaghetti that will fit inside the jar?

Pythagoras' Theorem in 3D

Week 4 Task 2

HM: 505, 506, 507

Exercise 1

Q9 A square-based pyramid has a base of side 4.8 cm and sloped edges all of length 11.2 cm. Find the vertical height of the pyramid from the centre of the base to the highest point.

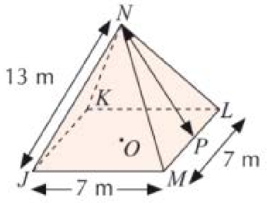
Q10 A square-based pyramid has a base of side 3.2 m and a vertical height of 9.2 m. Find the length of the sloped edges of the pyramid, given they are all equal.

Q11 The square-based pyramid *JKLMN* is shown on the right. *O* is the centre of the square base, directly below *N*. *P* is the midpoint of *LM*.

a) Find the length *NP*.

b) Find the length *OJ*.

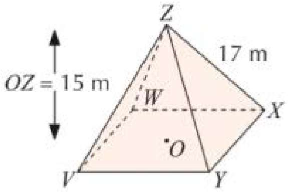
c) Find the length *ON*.



Q12 The square-based pyramid *VWXYZ* is shown on the right. Point *O*, the centre of the square *VWXY*, is directly below *Z*.

a) Find the length *VX*.

b) Hence find the area of the square *VWXY*.



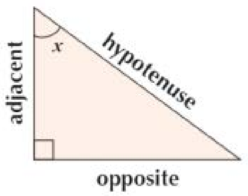
Trigonometry - Sin, Cos and Tan

Week 4 Task 3

HM: 508, 509, 510, 511

Trigonometry can be used to find lengths or angles in **right-angled triangles**. For a given angle x (as shown on the diagram on the right):

- The side opposite the right angle is the **hypotenuse**.
- The side opposite the given angle is the **opposite**.
- The side between the given angle and the right-angle is the **adjacent**.



The three sides of a right-angled triangle are linked by the following formulas:

$$\sin x = \frac{\text{opp}}{\text{hyp}}, \qquad \cos x = \frac{\text{adj}}{\text{hyp}}, \qquad \tan x = \frac{\text{opp}}{\text{adj}}$$

Tip: Remember 'SOH CAH TOA' to help you decide which formula you need to use.

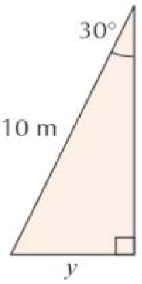
If you're given **one angle** and **one side**, you can use trigonometry to find an **unknown side length**. Look at the side you've been **given** and the side you **want to find** to decide which formula to use — e.g. if you had the **hypotenuse** and wanted to find the **adjacent**, you'd use the **cos** formula as it contains these two sides. **Substitute** the values you know into the appropriate formula and **rearrange** it to find the length you want.

Trigonometry - Sin, Cos and Tan

Example 1

Find the length of side *y*.

1. You're given the hypotenuse and asked to find the opposite, so use the formula for sin *x* and substitute in the values you know. $\sin x = \frac{\text{opp}}{\text{hyp}}$ $\sin 30° = \frac{y}{10}$
2. Rearrange the formula to find *y*. $y = 10 \sin 30°$
3. Input '10 sin 30' into your calculator and press '=' to find the value of *y*. $\mathbf{y = 5\ m}$

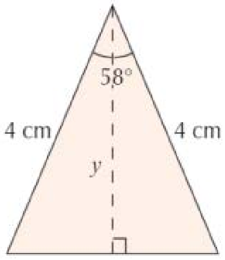


Trigonometry - Sin, Cos and Tan

Example 2

Find the height of the isosceles triangle shown. Give your answer correct to 3 significant figures.

1. Create a right-angled triangle by splitting the triangle in half. Divide the angle by 2 to find the angle in your right-angled triangle. $58 \div 2 = 29°$
2. Now you have the hypotenuse, and you need to find the adjacent, so use the formula for cos *x*. $\cos x = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 29° = \frac{y}{4}$
3. Rearrange the formula to find *y*. $y = 4 \cos 29°$
4. Use your calculator to find the value of *y*. $y = 3.498... =$ **3.50 cm** (3 s.f.)

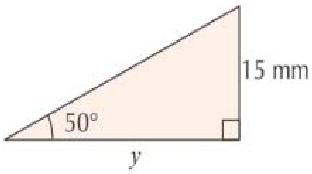


Trigonometry - Sin, Cos and Tan

Week 4 Task 3
HM: 508, 509, 510

Example 3

Find the length of side y. Give your answer correct to 3 significant figures.



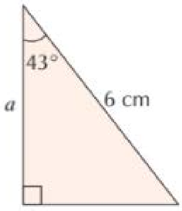
1. You're given the opposite and asked to find the adjacent, so use the formula for tan x.
   $\tan x = \frac{\text{opp}}{\text{adj}}$
2. Rearrange the formula to find y — this time, y is on the bottom of the fraction, so the rearrangement is slightly different.
   $\tan 50° = \frac{15}{y}$
   $y \times \tan 50° = 15 \Rightarrow y = 15 \div \tan 50°$
3. Use your calculator to find the value of y.
   $y = 12.586... =$ **12.6 mm** (3 s.f.)

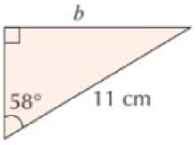
Trigonometry - Sin, Cos and Tan

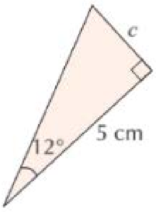
Week 4 Task 3
HM: 508, 509, 510

Exercise 1

Q1 Find the lengths of the sides marked with letters below. Give your answers to 3 significant figures.

a)


b)


c)


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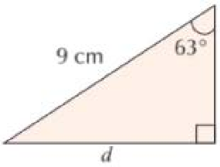
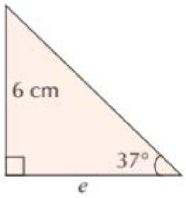
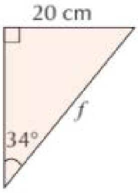
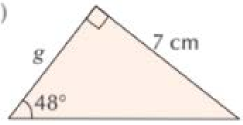


e)


f)


g)


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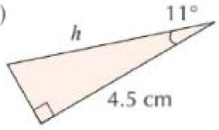
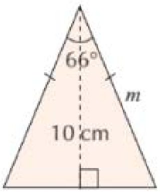
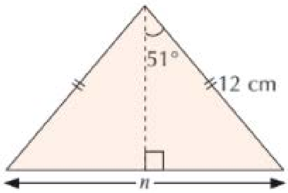


Q2 Find the lengths marked with letters in the triangles below.
Give your answers to 3 significant figures.

a)


b)


Trigonometry - Sin, Cos and Tan

Week 4 Task 3
HM: 511, 512

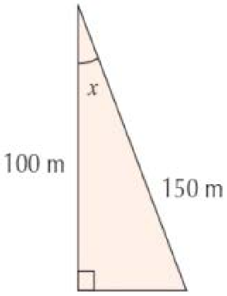
You can also use the trig formulas to find an **angle** if you know two side lengths. You have to use the **inverse functions** of sin, cos and tan (written **sin⁻¹**, **cos⁻¹** and **tan⁻¹**), which return an **angle**. To find an angle, work out which formula you need from the sides you're given as before, then **substitute** in the known values — this will give you a **fraction** on the right-hand side, e.g. $\sin x = \frac{1}{2}$. Take the **inverse trig function** of the fraction to get the angle — so here you'd do $x = \sin^{-1}\left(\frac{1}{2}\right) = 30°$.

Trigonometry - Sin, Cos and Tan

Week 4 Task 3
HM: 511, 512

Example 4

Find the size of angle x. Give your answer correct to 1 decimal place.



1. You're given the adjacent and the hypotenuse, so use the formula for cos x. $\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{100}{150}$
2. Take the inverse of cos to find the angle. $x = \cos^{-1}\left(\frac{100}{150}\right)$
3. Input 'cos⁻¹(100 ÷ 150)' into your calculator and press '=' to find the value of x. $x = 48.189... =$ **48.2°** (1 d.p.)

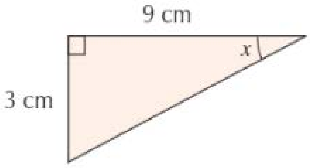
Trigonometry - Sin, Cos and Tan

Week 4 Task 3

HM: 511, 512

Example 5

Find the size of angle x. Give your answer correct to 1 decimal place.



1. You're given the opposite and the adjacent, so use the formula for tan x. $\tan x = \frac{\text{opp}}{\text{adj}} = \frac{3}{9}$
2. Take the inverse of tan to find the angle. $x = \tan^{-1}\left(\frac{3}{9}\right)$
3. Use your calculator to find the value of x. $x = 18.434... = \mathbf{18.4°}$ (1 d.p.)

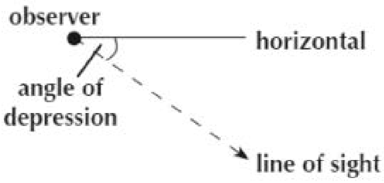
Trigonometry - Sin, Cos and Tan

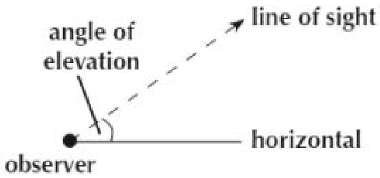
Week 4 Task 3

HM: 515

Trigonometry can be used to work out angles of **depression** and **elevation**.

The **angle of depression** is the angle between a **horizontal line** and the **line of sight** of an observer at the same level **looking down** — e.g. you could measure the angle of depression of someone looking down from a window.





The **angle of elevation** is the angle between a **horizontal line** and the **line of sight** of an observer at the same level **looking up** — e.g. the angle made looking up at a hovering helicopter.

For problems like this, use the information given to **draw** a right-angled triangle and use the formulas in the **same way** as usual. Remember to relate your answer to the **original context** of the problem.

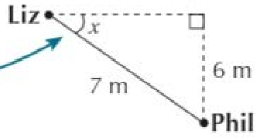
Trigonometry - Sin, Cos and Tan

Week 4 Task 3

HM: 515

Example 6

Liz holds one end of a 7 m paper chain out of her window. Phil stands in the garden below holding the other end to its full extent. Phil's end of the paper chain is 6 m vertically below Liz's end. Find the size of the angle of depression from Liz to Phil. Give your answer to 1 decimal place.



1. Use the information to draw a right-angled triangle — the angle of depression is the angle below the horizontal.	
2. You're given the hypotenuse and the opposite, so use the formula for sin x.	$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{6}{7}$
3. Take the inverse of sin to find the angle.	$x = \sin^{-1}\left(\frac{6}{7}\right)$
4. Use your calculator to find the value of x.	$x = 58.997... = \mathbf{59.0°}$ (1 d.p.)

Trigonometry - Sin, Cos and Tan

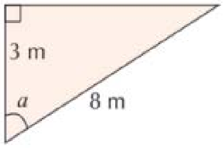
Week 4 Task 3

HM: 515

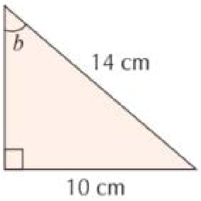
Exercise 2

Q1 Find the size of the angles marked with letters. Give your answers to 1 decimal place.

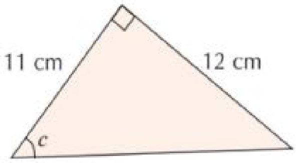
a)



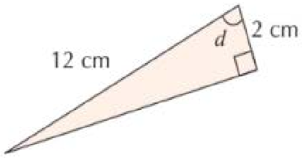
b)



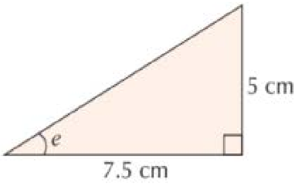
c)



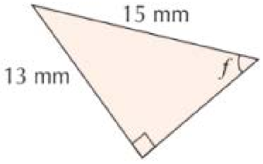
d)



e)



f)



Trigonometry - Sin, Cos and Tan

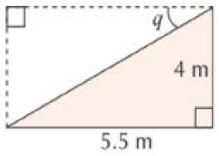
Week 4 Task 3

HM: 515

Exercise 2

Q2 Melissa is building slides for an adventure playground.

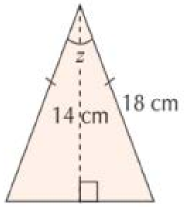
a) The first slide she builds has an 8 m high vertical ladder and a slide of length 24 m. Find *m*, the slide's angle of elevation, to 1 d.p.



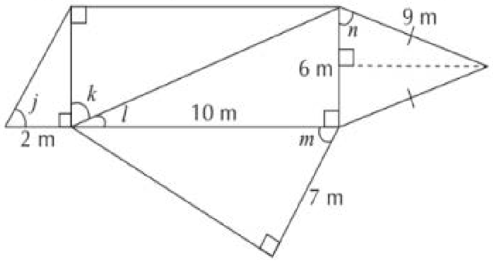
b) A second slide has a 4 m vertical ladder, and the base of the slide reaches the ground 5.5 m from the base of the ladder as shown. Find *q*, the angle of depression at the top of the second slide, to 1 d.p.

Q3 Find the angles marked with letters in the following diagrams. Give your answers correct to 1 d.p.

a)



b)

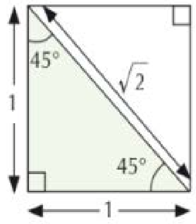


Trigonometry - Sin, Cos and Tan

Week 4 Task 3

HM: n/a

The sin, cos and tan of some angles have **exact values**. You need to either **remember** the values or know how to work them out **without a calculator**. The first of these angles is **45°**.



Start by drawing a **square** with sides of length **1**. Then **split** the square down its **diagonal** to create two **right-angled triangles**. By Pythagoras' theorem, the length of the hypotenuse is $\sqrt{1^2 + 1^2} = \sqrt{\mathbf{2}}$. Since the triangle was formed by **bisecting** 90° angles, the two acute angles in the triangle are 90° ÷ 2 = **45°**. Then use the trig formulas to find the sin, cos and tan of 45° — e.g. $\sin 45° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$.

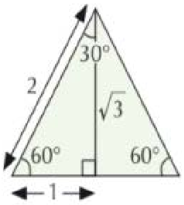
$$\sin 45° = \frac{1}{\sqrt{2}} \qquad \cos 45° = \frac{1}{\sqrt{2}} \qquad \tan 45° = 1$$

Trigonometry - Sin, Cos and Tan

Week 4 Task 3

HM: n/a

To find sin, cos and tan of **30°** and **60°**, start with an **equilateral triangle** with sides of length **2**. The interior angles are **60°** (see p.252). **Split** the triangle in half using a **perpendicular** line from one vertex to the opposite side, leaving two **right-angled triangles** with acute angles of **60°** and 60° ÷ 2 = **30°**. By Pythagoras' theorem, the perpendicular height is $\sqrt{2^2 - 1^2} = \sqrt{3}$. Then use the trig formulas to get the following results.



$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \sqrt{3}$

$\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\tan 30° = \frac{1}{\sqrt{3}}$

You **can't** use triangles for **0°** or **90°** but you do need to know their trig values:

sin 0° = 0 **cos 0° = 1** **tan 0° = 0**

sin 90° = 1 **cos 90° = 0** **tan 90° = undefined**

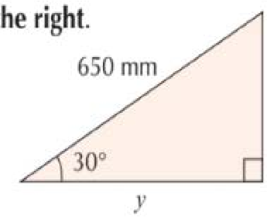
Trigonometry - Sin, Cos and Tan

Week 4 Task 3

HM: n/a

Example 7

Without using a calculator, find the exact length of side *y* on the triangle on the right.



1. You're given the hypotenuse and want to find the adjacent, so use the formula for cos *x*.
2. Substitute in the values you know and rearrange the formula to make *y* the subject.
3. Replace cos 30° with its exact trig value and work through the formula to find *y*.

$\cos x = \frac{\text{adj}}{\text{hyp}}$

$\cos 30° = \frac{y}{650}$

$y = 650 \times \cos 30°$

$= 650 \times \frac{\sqrt{3}}{2} = \mathbf{325\sqrt{3}\ mm}$

Trigonometry - Sin, Cos and Tan

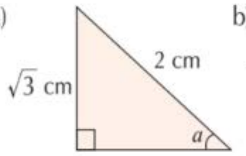
Week 4 Task 3

HM: n/a

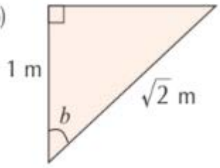
Exercise 3

Q1 Find the size of the angles marked with letters.

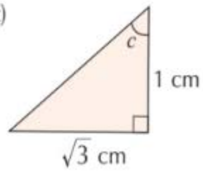
a)



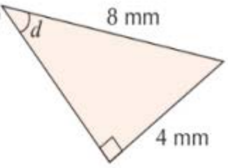
b)



c)

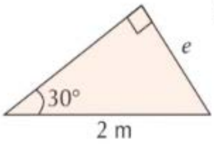


d)

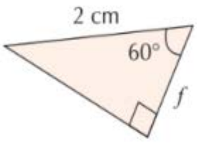


Q2 Find the exact length of the sides marked with letters.

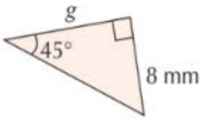
a)



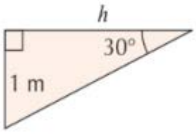
b)



c)



d)

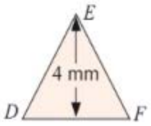


Q3 Show that:

a) $\tan 45° + \sin 60° = \frac{2+\sqrt{3}}{2}$ b) $\sin 45° + \cos 45° = \sqrt{2}$ c) $\tan 30° + \tan 60° = \frac{4\sqrt{3}}{3}$

Q4 Triangle *ABC* is isosceles. $AC = 7\sqrt{2}$ cm and angle $ABC = 90°$.
What is the exact length of side *AB*?

Q5 Triangle *DEF* is an equilateral triangle with a perpendicular height of 4 mm.
What is the exact side length of the triangle? Give your answer in its simplest form.



The Sine and Cosine Rules

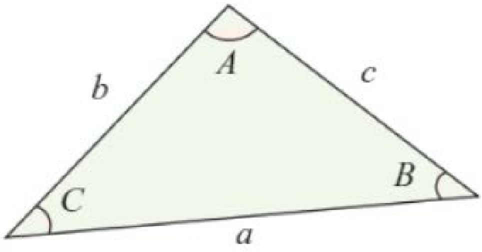
Week 4 Task 4

HM: 521, 522, 523, 524, 525

To use trigonometry in triangles that **aren't** right-angled, you must first **label** the sides and angles properly, like in the diagram on the right — side *a* must always be **opposite** angle *A* etc. You use **lower case letters** for the **sides** and **upper case letters** for the **angles**.

You can then use the **sine rule**, which is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

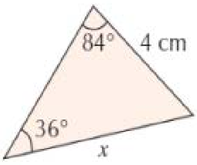


The Sine and Cosine Rules

Week 4 Task 4
HM: 521, 522, 523, 524, 525

Example 1

Find the length of side x. Give your answer to 3 significant figures.



1. Substitute the values from the diagram into the sine rule. $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{4}{\sin 36°} = \frac{x}{\sin 84°}$
2. Rearrange to make x the subject. $x = \frac{4\sin 84°}{\sin 36°}$
3. Use your calculator to find the value of x. $x = \mathbf{6.77\ cm}$ (3 s.f.)

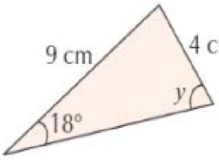
Tip: Label the triangle using a, A, b and B if you need to.

The Sine and Cosine Rules

Week 4 Task 4
HM: 521, 522, 523, 524, 525

Example 2

Find the size of the acute angle y below. Give your answer to 1 decimal place.



1. Substitute in the values from the diagram. The second version of the formula works best here, but the first would also work. $\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin y}{9} = \frac{\sin 18°}{4}$
2. Rearrange the equation. $\sin y = \frac{9\sin 18°}{4} \Rightarrow y = \sin^{-1}\left(\frac{9\sin 18°}{4}\right)$
3. Use the inverse function to find y. $y = \mathbf{44.1°}$ (1 d.p.)

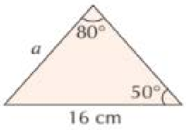
The Sine and Cosine Rules

Week 4 Task 4
HM: 521, 522, 523, 524, 525

Exercise 1

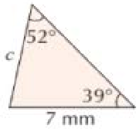
Q1 Find the lengths of the sides marked with letters below. Give your answers to 3 significant figures.

a)



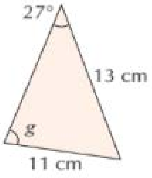
b)

c)

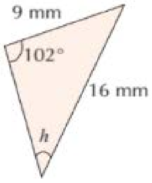


Q2 For each triangle below, find the acute angle marked with a letter. Give your answers to 1 d.p.

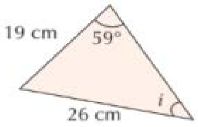
a)



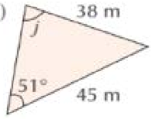
b)



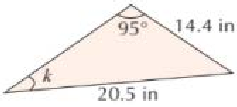
c)



d)



e)



f)

Q3 The triangle XYZ is such that angle $YXZ = 55°$, angle $XYZ = 40°$ and length $YZ = 83$ m. Find:

a) length XZ, to 3 s.f.

b) length XY, to 3 s.f.

Q4 A triangular piece of metal PQR is such that angle $RPQ = 61°$, length $QR = 13.1$ mm and length $PQ = 7.2$ mm. Find the size of the acute angle PQR, correct to 1 decimal place.

Q5 Point B is 13 km north of point A. Point C lies 19 km from point B, on a bearing of 052° from A. Find the bearing of C from B, to the nearest degree.

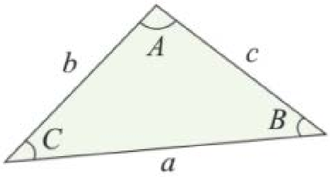
The Sine and Cosine Rules

Week 4 Task 4
HM: 526, 527, 528, 529, 530

The **cosine rule** can also be used to find unknown angles and sides.

For any triangle labelled as shown in the diagram, the cosine rule is:

$$a^2 = b^2 + c^2 - 2bc\cos A$$



This can be rearranged to give $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

This version is useful when you're trying to find an **angle**.

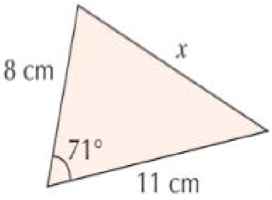
The Sine and Cosine Rules

Week 4 Task 4

HM: 526, 527, 528, 529, 530

Example 3

Find the length of side x. Give your answer to 3 significant figures.



1. Substitute the values from the diagram into the cosine rule — x is side a since it's opposite the known angle (A).

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 8^2 + 11^2 - (2 \times 8 \times 11)\cos 71°$$

2. Work it through to find x^2.

$$x^2 = 127.70...$$

3. Take the square root to find the value of x.

$$x = \sqrt{127.70...} = \mathbf{11.3\ cm}\ (3\text{ s.f.})$$

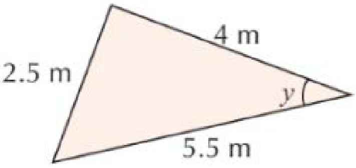
The Sine and Cosine Rules

Week 4 Task 4

HM: 526, 527, 528, 529, 530

Example 3

Find the size of angle y in the triangle on the right.
Give your answer to 1 decimal place.



1. Use the second version of the cosine rule to find the angle.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

2. Substitute the values from the diagram into the formula — make the side opposite the angle a.

$$\cos y = \frac{5.5^2 + 4^2 - 2.5^2}{2 \times 5.5 \times 4}$$

3. Use the inverse cos function to find the value of y.

$$y = \cos^{-1}\left(\frac{5.5^2 + 4^2 - 2.5^2}{2 \times 5.5 \times 4}\right)$$
$$= \mathbf{24.6°}\ (1\text{ d.p.})$$

The Sine and Cosine Rules

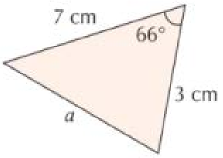
Week 4 Task 4

HM: 526, 527, 528, 529, 530

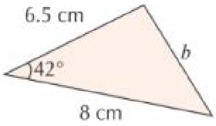
Exercise 2

Q1 Use the cosine rule to find the lengths of the sides marked with letters. Give your answers to 3 s.f.

a)



b)



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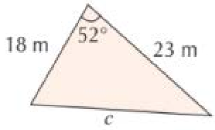

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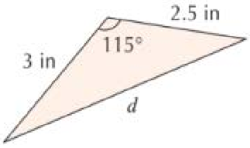

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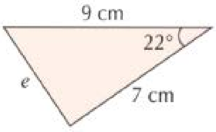

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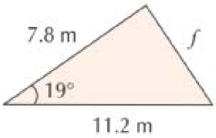
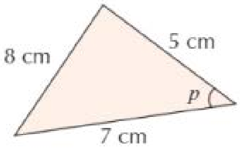


Q2 Use the cosine rule to find the sizes of the angles marked with letters.
Give your answers to 1 d.p.

a)



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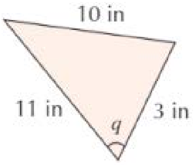

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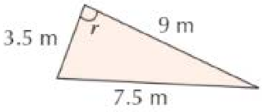

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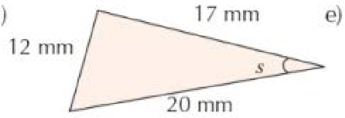

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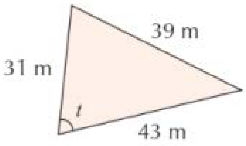

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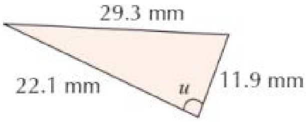


The Sine and Cosine Rules

Week 4 Task 4

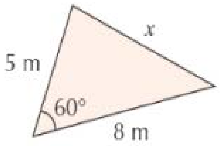
HM: 526, 527, 528, 529, 530

Exercise 2

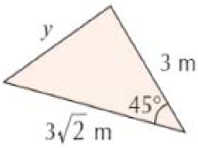
Q3 A triangular sign *XYZ* is such that *XY* = 67 cm, *YZ* = 78 cm and *XZ* = 99 cm.
Find the size of the angle *XYZ*, correct to 1 d.p.

Q4 Find the exact values of the letters in each of these triangles.

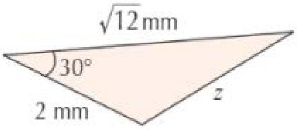
a)



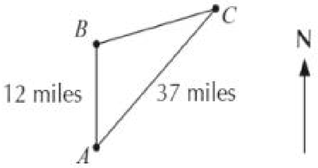
b)



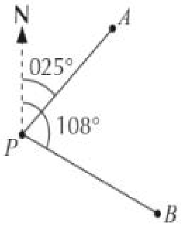
c)



Q5 Village *B* is 12 miles north of village *A*. Village *C* is 37 miles north-east of village *A*. Find the direct distance between village *B* and village *C*, correct to 3 s.f.



Q6 Two ramblers set off walking from point *P*.
The first rambler walks for 2 km on a bearing of 025° to point *A*.
The second rambler walks for 3 km on a bearing of 108° to point *B*.
Find the direct distance between *A* and *B*, correct to 3 s.f.



The Sine and Cosine Rules

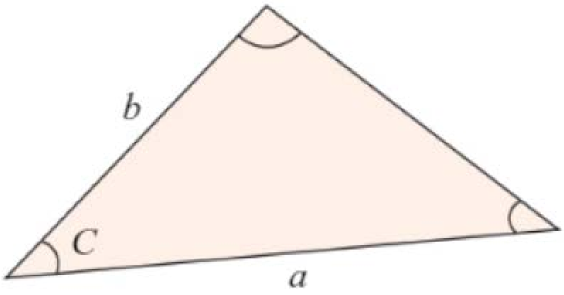
Week 4 Task 4

HM: 516, 517, 518, 519

To find the **area** of a triangle, use the formula: $\text{Area} = \frac{1}{2}ab\sin C$

To use this formula, you need to know **two sides and the angle between them.** If you don't, you might have to use the sine and cosine rules first to find the information you need.

You can also use the formula to find the **size of an angle** or **length of a side** when you know the area — just **substitute** in the values you know and **rearrange** the formula to make the angle or side you want the subject.



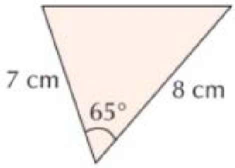
The Sine and Cosine Rules

Week 4 Task 4

HM: 516, 517, 518, 519

Example 5

Find the area of this triangle. Give your answer to 3 significant figures.



1. Substitute the values from the diagram into the formula.
2. Use your calculator to find the area. Make sure you use the correct units.

$$\text{Area} = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 7 \times 8 \times \sin 65°$$
$$= \mathbf{25.4\ cm^2}\ (3\text{ s.f.})$$

The Sine and Cosine Rules

Week 4 Task 4

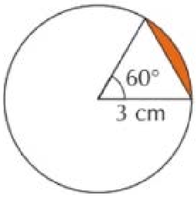
HM: 516, 517, 518, 519

Example 6

A segment is formed in a sector with angle 60° in a circle of radius 3 cm. Find the area of the segment. Give your answer to 3 significant figures.

1. Work out the area of the sector.

Area of sector $= \frac{60}{360} \times \pi \times 3^2 = \frac{3\pi}{2}$ cm^2



2. The triangle is isosceles (two sides are radii) so you know two sides and the angle between them. Use this information to work out the area.

Area of triangle $= \frac{1}{2} \times 3 \times 3 \times \sin 60°$

$= \frac{9\sqrt{3}}{4}$ cm^2

3. Subtract the area of the triangle from the area of the sector to give the area of the segment.

Area of segment $= \frac{3\pi}{2} - \frac{9\sqrt{3}}{4}$

$=$ **0.815 cm^2** (3 s.f.)

The Sine and Cosine Rules

Week 4 Task 4

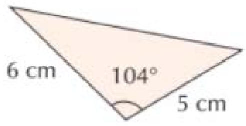
HM: 516, 517, 518, 519

Exercise 3

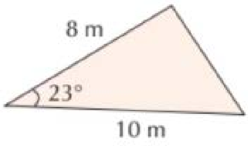
Give all your answers to the questions in this exercise to 3 significant figures.

Q1 Find the area of each of the following triangles.

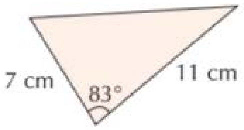
a)



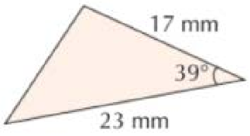
b)



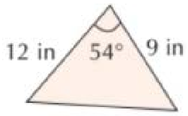
c)



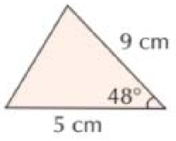
d)



e)



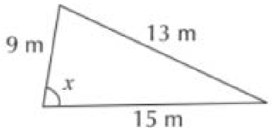
f)

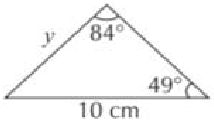


Q2 Find the area of the segment formed in a sector with angle 67° in a circle of radius 4.5 cm.

Q3 A field in the shape of an equilateral triangle has sides of length 32 m. Find the area of the field.

Q4 For the triangle on the right, use the cosine rule to calculate x and hence find the area of the triangle.





Q5 For the triangle on the left, use the sine rule to calculate y and hence find the area of the triangle.

Trigonometry in 3D

Week 4 Task 5

HM: 856, 857, 858, 859, 860

You can use **trigonometry** to find lengths and angles in **3D shapes** by creating triangles within them. You can use sin, cos and tan for **right-angled triangles** and the **sine** or **cosine rules** for other triangles. You might need to use **Pythagoras' theorem** too — see p.321. The formulas are used in the same way as for 2D shapes, but you might have to use them **multiple** times to find what you're looking for. It's often a good idea to **sketch** the triangles as you go along to keep track of what you're doing.

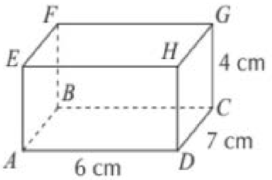
Trigonometry in 3D

Week 4 Task 5

HM: 856, 857, 858, 859, 860

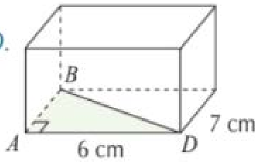
Example 1

Find the angle *BDF* in the cuboid shown on the right.



1. Use Pythagoras' theorem on the triangle *ABD* to find the length *BD*.

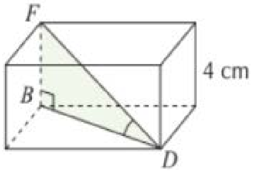
$BD^2 = AD^2 + AB^2$
$= 6^2 + 7^2 = 85$
$BD = \sqrt{85}$ cm



2. *BDF* forms a right-angled triangle and you know the opposite and adjacent sides to angle *BDF*, so use the tan formula.

$\tan BDF = \frac{\text{opp}}{\text{adj}} = \frac{FB}{BD} = \frac{4}{\sqrt{85}}$

$BDF = \tan^{-1}\left(\frac{4}{\sqrt{85}}\right)$
$= \mathbf{23.5°}$ (1 d.p.)



Tip: There may be alternative triangles you can use to find the angle you want.

Trigonometry in 3D

Week 4 Task 5

HM: 856, 857, 858, 859, 860

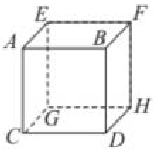
Exercise 1

Q1 The cube shown has sides of length 3 m. Find:

a) the exact length *AF*

b) the exact length *FC*

c) the angle *AHC*, to 1 d.p.



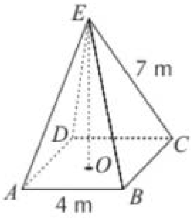
Q2 *ABCDE* is a square-based pyramid. The centre of the base is *O*, and *E* lies directly above *O*. Find:

a) the angle *BCE*, to 1 d.p

b) the angle *AEB*, to 1 d.p.

c) the exact vertical height *EO*

d) the angle *AEO*, to 1 d.p.

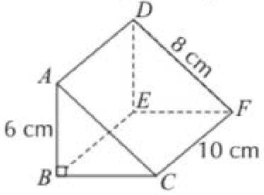


Q3 For the triangular prism shown, find:

a) the angle *EDF*, to 1 d.p.

b) the exact length *DC*

c) the angle *DCE*, to 1 d.p.



Trigonometry in 3D

Week 4 Task 5

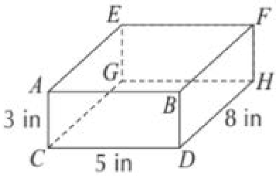
HM: 856, 857, 858, 859, 860

Exercise 1

Q4 For the cuboid shown, find:

a) the exact length *AH*

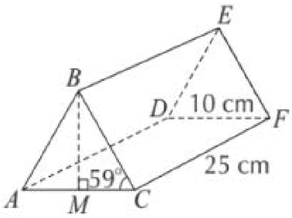
b) the angle *EDG*, to 1 d.p.



Q5 For the triangular prism shown, where *M* is the midpoint of *AC*, find:

a) the perpendicular height *BM*, to 3 s.f.

b) the length *EM*, to 3 s.f.

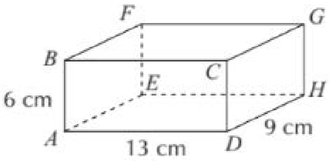


Trigonometry in 3D

Week 4 Task 5

HM: 856, 857, 858, 859, 860

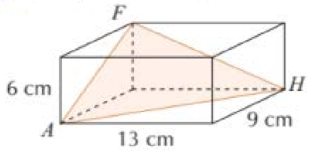
Example 2



Find the size of angle AFH in the cuboid shown on the right.

1. Use Pythagoras' theorem to find the lengths AF, AH and FH.

$AF^2 = 6^2 + 9^2 = 117 \Rightarrow AF = \sqrt{117}$
$AH^2 = 13^2 + 9^2 = 250 \Rightarrow AH = \sqrt{250}$
$FH^2 = 6^2 + 13^2 = 205 \Rightarrow FH = \sqrt{205}$



2. AFH isn't a right-angled triangle but you know all 3 sides so use the cosine rule.

$$\cos AFH = \frac{AF^2 + FH^2 - AH^2}{2 \times AF \times FH} = \frac{117 + 205 - 250}{2 \times \sqrt{117} \times \sqrt{205}}$$

3. Take the inverse of cos to find the angle.

$$AFH = \cos^{-1}\left(\frac{117 + 205 - 250}{2 \times \sqrt{117} + \sqrt{205}}\right) = \mathbf{76.6°} \text{ (1 d.p.)}$$

Trigonometry in 3D

Week 4 Task 5

HM: 856, 857, 858, 859, 860

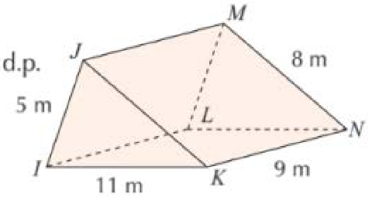
Exercise 2

Q1 The diagram shows the triangular prism $IJKLMN$.

a) Use the cosine rule to find the size of angle JIK, correct to 1 d.p.

b) Hence find the area of triangle IJK, correct to 1 d.p.

c) Hence find the volume of the triangular prism $IJKLMN$, to the nearest m³.



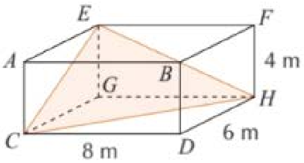
Q2 The diagram shows the triangle CEH drawn inside the cuboid $ABCDEFGH$.

a) Find the exact length CE.

b) Find the exact length CH.

c) Find the exact length EH.

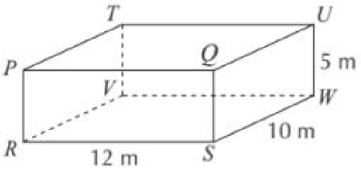
d) Hence use the cosine rule to find the size of angle ECH, correct to 1 d.p.



Q3 The diagram shows the cuboid $PQRSTUVW$.

a) Use Pythagoras and the cosine rule to find the size of angle PSU, correct to 1 d.p.

b) Hence find the area of triangle PSU, correct to 1 d.p.



Week 4 Assessment

Question 1

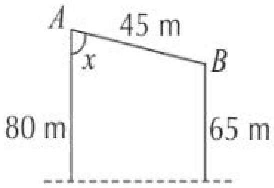
A ladder leans against the side of a vertical tower. It reaches a window 20 m above the ground.
The base of the ladder is placed 8 m away from the bottom of the tower.
What is the angle of elevation? Give your answer correct to 1 decimal place.

[2 marks]

Week 4 Assessment

Question 2

A taut zip line of length 45 m goes between platforms A and B. Platform A is 80 m above the horizontal ground, and platform B is 65 m above the ground, as shown below.



Find x, the angle the wire forms with the vertical at platform A.
Give your answer correct to 1 decimal place.

[2 marks]

Week 4 Assessment

Question 3

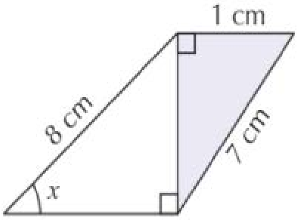
The points P and R have coordinates $P(1, 3)$ and $R(7, 8)$.
Find the distance between P and R, correct to 3 significant figures.

[2 marks]

Week 4 Assessment

Question 4

The shape shown below is made up of two right-angled triangles.
Find the size of angle x.

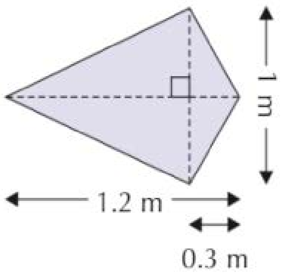


[3 marks]

Week 4 Assessment

Question 5

Rahim wants to put a gold ribbon around the edges of the kite shown below.



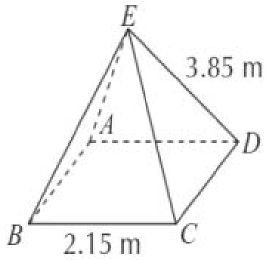
Ribbon is sold in lengths of 10 cm. What length of ribbon should he buy? PROBLEM SOLVING

[3 marks]

Week 4 Assessment

Question 6

The pyramid *ABCDE* has a square base of side length 2.15 m. Point *E* is directly above the centre of the square *ABCD*. The edge *ED* is 3.85 m long. Work out the angle between *ED* and the base, giving your answer to 1 decimal place.



[3 marks]

Week 4 Assessment

Question 7

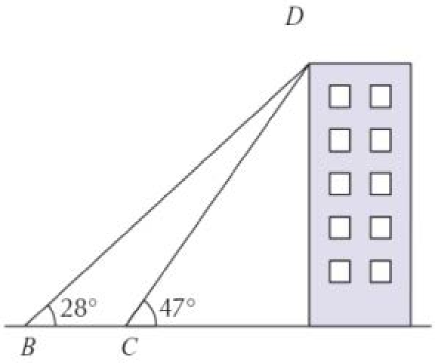
An aeroplane takes off from a flat runway and flies for 675 m in a straight line, after which time it is 250 m vertically above the horizontal ground. Work out the angle of elevation of the plane, to 3 significant figures.

[2 marks]

Week 4 Assessment

Question 8

Bianca (*B*) and Caleb (*C*) are standing 25 m apart in a straight line directly facing a vertical tower block, as shown in the diagram. The ground is horizontal. From where Bianca is standing, the angle of elevation from the ground to the top of the tower block is 28°. From where Caleb is standing, the angle of elevation from the ground to the top of the tower block is 47°.



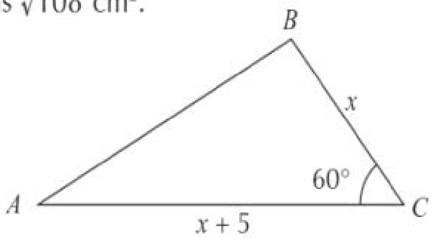
Work out the height of the tower block, giving your answer to three significant figures.

[4 marks]

Week 4 Assessment

Question 9

The diagram shows a triangle ABC where angle $ACB = 60°$, $AC = x + 5$ cm and $BC = x$ cm. The area of the triangle is $\sqrt{108}$ cm^2.



Find the length of AB.

[6 marks]

Week 1 answers

Model Solutions on Indices

Exercise 1

1 a) 2^5 b) 7^7 c) $3^2x^3y^2$

2 a) 10^3 b) 10^7 c) 10^8

3 a) 81 b) 256 c) 59 049 d) 59 049

e) 768 f) 40 g) 512 h) 537 289

4 a) $\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \mathbf{\frac{1}{4}}$ b) $\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \mathbf{\frac{1}{8}}$

c) $\left(\frac{1}{4}\right)^2 = \frac{1^2}{4^2} = \mathbf{\frac{1}{16}}$ d) $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \mathbf{\frac{4}{9}}$

e) $\left(\frac{3}{10}\right)^2 = \frac{3^2}{10^2} = \mathbf{\frac{9}{100}}$ f) $\left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \mathbf{\frac{27}{8}}$

g) $\left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4} = \mathbf{\frac{625}{81}}$ h) $\left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \mathbf{\frac{64}{27}}$

Exercise 2

1 a) $3^2 \times 3^6 = 3^{2+6} = \mathbf{3^8}$

b) $10^7 \div 10^3 = 10^{7-3} = \mathbf{10^4}$

c) $a^6 \times a^4 = a^{6+4} = \boldsymbol{a^{10}}$

d) $(4^3)^3 = 4^{3 \times 3} = \mathbf{4^9}$

e) $8^6 \div 8^1 = 8^{6-1} = \mathbf{8^5}$

f) $7 \times 7^6 = 7^1 \times 7^6 = 7^{1+6} = \mathbf{7^7}$

g) $(c^5)^4 = c^{5 \times 4} = \boldsymbol{c^{20}}$

h) $\frac{b^8}{b^5} = b^8 \div b^5 = b^{8-5} = \boldsymbol{b^3}$

i) $f^{75} \div f^0 = f^{75-0} = \boldsymbol{f^{75}}$

j) $\frac{20^{228}}{20^{210}} = 20^{228-210} = \mathbf{20^{18}}$

k) $(g^{11})^8 = g^{11 \times 8} = \boldsymbol{g^{88}}$

l) $(14^7)^d = 14^{7 \times d} = \boldsymbol{14^{7d}}$

2 a) 8 − 3 = ■ ⇒ ■ = **5**

b) ■ + 10 = 12 ⇒ ■ = 12 − 10 = **2**

c) 10 × 4 = ■ ⇒ ■ = **40**

d) 6 × ■ = 24 ⇒ ■ = 24 ÷ 6 = **4**

e) ■ × 10 = 30 ⇒ ■ = 30 ÷ 10 = **3**

f) 7 + ■ = 13 ⇒ ■ = 13 − 7 = **6**

g) ■ − 6 = 7 ⇒ ■ = 7 + 6 = **13**

h) 14 − ■ = 7 ⇒ ■ = 14 − 7 = **7**

Exercise 2

3 a) $3^2 \times 3^5 \times 3^7 = 3^{2+5+7} = \mathbf{3^{14}}$

b) $5^4 \times 5 \times 5^8 = 5^{4+1+8} = \mathbf{5^{13}}$

c) $(p^6)^2 \times p^5 = p^{6 \times 2 + 5} = p^{12+5} = \boldsymbol{p^{17}}$

d) $(9^4 \times 9^3)^5 = 9^{(4+3) \times 5} = 9^{7 \times 5} = \mathbf{9^{35}}$

e) $7^3 \times 7^5 \div 7^6 = 7^{3+5-6} = \mathbf{7^2}$

f) $8^3 \div 8^9 \times 8^7 = 8^{3-9+7} = 8^1 = \mathbf{8}$

g) $(12^8 \div 12^4)^3 = 12^{(8-4) \times 3} = 12^{4 \times 3} = \mathbf{12^{12}}$

h) $(q^3)^6 \div q^4 = q^{3 \times 6 - 4} = q^{18-4} = \boldsymbol{q^{14}}$

4 a) $\frac{3^4 \times 3^5}{3^6} = \frac{3^9}{3^6} = 3^{9-6} = \mathbf{3^3}$

b) $\frac{s^8 \times s^4}{s^3 \times s^6} = \frac{s^{12}}{s^9} = s^{12-9} = \boldsymbol{s^3}$

c) $\left(\frac{6^3 \times 6^9}{6^7}\right)^3 = \left(\frac{6^{12}}{6^7}\right)^3 = (6^{12-7})^3$
$= (6^5)^3 = 6^{5 \times 3} = \mathbf{6^{15}}$

d) $\frac{2^5 \times 2^5}{(2^3)^2} = \frac{2^{10}}{2^6} = 2^{10-6} = \mathbf{2^4}$

e) $\frac{5^5 \times 5^5}{5^8 \div 5^3} = \frac{5^{10}}{5^5} = 5^{10-5} = \mathbf{5^5}$

f) $\frac{10^8 \div 10^3}{10^4 \div 10^4} = \frac{10^5}{10^0} = 10^{5-0} = \mathbf{10^5}$

You could also write $10^0 = 1$ and then use the fact that anything divided by 1 is itself.

g) $\frac{(t^6 \div t^3)^4}{t^9 \div t^4} = \frac{(t^3)^4}{t^5} = \frac{t^{12}}{t^5} = t^{12-5} = \boldsymbol{t^7}$

h) $\frac{(8^5)^7 \div 8^{12}}{8^6 \times 8^{10}} = \frac{8^{35} \div 8^{12}}{8^{16}} = \frac{8^{23}}{8^{16}} = 8^{23-16} = \mathbf{8^7}$

Exercise 2

5 a) (i) $4 = \mathbf{2^2}$

(ii) $4^5 = (2^2)^5 = 2^{2 \times 5} = \mathbf{2^{10}}$

(iii) $2^3 \times 4^5 = 2^3 \times 2^{10} = 2^{3 + 10} = \mathbf{2^{13}}$

b) (i) $9 \times 3^3 = 3^2 \times 3^3 = 3^{2 + 3} = \mathbf{3^5}$

(ii) $5 \times 25 \times 125 = 5 \times 5^2 \times 5^3$

$= 5^{1 + 2 + 3} = \mathbf{5^6}$

(iii) $16 \times 2^6 = 2^4 \times 2^6 = 2^{4 + 6} = 2^{10}$

$= 2^{2 \times 5} = (2^2)^5 = \mathbf{4^5}$

Exercise 3

1 a) $4^{-1} = \frac{1}{4^1} = \mathbf{\frac{1}{4}}$ b) $2^{-2} = \frac{1}{2^2} = \mathbf{\frac{1}{4}}$

c) $3^{-3} = \frac{1}{3^3} = \mathbf{\frac{1}{27}}$

d) $2 \times 3^{-1} = 2 \times \frac{1}{3} = \mathbf{\frac{2}{3}}$

2 a) $\mathbf{5^{-1}}$ b) $\mathbf{11^{-1}}$ c) $\mathbf{3^{-2}}$ d) $\mathbf{2^{-7}}$

3 a) $\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^{1} = \mathbf{2}$

b) $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^{2} = 3^2 = \mathbf{9}$

c) $\left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^{3} = \frac{2^3}{5^3} = \mathbf{\frac{8}{125}}$

d) $\left(\frac{7}{10}\right)^{-2} = \left(\frac{10}{7}\right)^{2} = \frac{10^2}{7^2} = \mathbf{\frac{100}{49}}$

Exercise 4

1 a) $5^4 \times 5^{-2} = 5^{4+(-2)} = \mathbf{5^2}$

b) $g^6 \div g^{-6} = g^{6-(-6)} = \mathbf{g^{12}}$

c) $2^{16} \div \frac{1}{2^4} = 2^{16} \div 2^{-4} = 2^{16-(-4)} = \mathbf{2^{20}}$

d) $k^{10} \times k^{-6} \div k^0 = k^{10+(-6)-0} = \mathbf{k^4}$

e) $\left(\frac{1}{p^4}\right)^5 = (p^{-4})^5 = p^{-4\times 5} = \mathbf{p^{-20}}$

f) $\left(\frac{l^{-5}}{l^6}\right)^{-3} = (l^{-5-6})^{-3} = (l^{-11})^{-3} = l^{-11\times -3} = \mathbf{l^{33}}$

g) $\frac{n^{-4}\times n}{(n^{-3})^6} = \frac{n^{-3}}{n^{-18}} = n^{-3-(-18)} = \mathbf{n^{15}}$

h) $\left(\frac{10^7\times 10^{-11}}{10^9\div 10^4}\right)^{-5} = \left(\frac{10^{-4}}{10^5}\right)^{-5} = (10^{-9})^{-5} = \mathbf{10^{45}}$

2 a) (i) $\mathbf{\frac{1}{100}}$ (ii) $\mathbf{\frac{1}{10^2}}$ (iii) $\mathbf{10^{-2}}$

b) (i) $\mathbf{10^{-1}}$ (ii) $\mathbf{10^{-8}}$ (iii) $\mathbf{10^{-4}}$ (iv) $\mathbf{10^0}$

3 a) $3^2 \times 5^{-2} = 3^2 \times \frac{1}{5^2} = \frac{3^2}{5^2} = \mathbf{\frac{9}{25}}$

b) $2^{-3} \times 7^1 = \frac{1}{2^3} \times 7 = \frac{7}{2^3} = \mathbf{\frac{7}{8}}$

c) $\left(\frac{1}{2}\right)^{-2} \times \left(\frac{1}{3}\right)^2 = 2^2 \times \frac{1}{3^2} = \mathbf{\frac{4}{9}}$

d) $6^{-4} \div 6^{-2} = 6^{-4-(-2)} = 6^{-2} = \frac{1}{6^2} = \mathbf{\frac{1}{36}}$

e) $(-9)^2 \times (-5)^{-3} = (-9)^2 \times \frac{1}{(-5)^3} = \mathbf{-\frac{81}{125}}$

f) $8^{-5} \times 8^3 \times 3^3 = 8^{-2} \times 3^3 = \frac{3^3}{8^2} = \mathbf{\frac{27}{64}}$

g) $10^{-5} \div 10^6 \times 10^4 = 10^{-7}$

$= \frac{1}{10^7} = \mathbf{\frac{1}{10\,000\,000}}$

h) $\left(\frac{3}{4}\right)^{-1} \div \left(\frac{1}{2}\right)^{-3} = \frac{4}{3} \div 2^3 = \frac{4}{3\times 2^3}$

$= \frac{4}{3\times 8} = \frac{4}{24} = \mathbf{\frac{1}{6}}$

Exercise 5

1 a) $\sqrt[5]{a}$ b) $(\sqrt[5]{a})^3$ c) $(\sqrt[5]{a})^2$ d) $(\sqrt{a})^5$

2 a) $64^{\frac{1}{2}} = \sqrt{64} = \mathbf{8}$ b) $64^{\frac{1}{3}} = \sqrt[3]{64} = \mathbf{4}$

c) $16^{\frac{1}{4}} = \sqrt[4]{16} = \mathbf{2}$

d) $1\,000\,000^{\frac{1}{2}} = \sqrt{1\,000\,000} = \mathbf{1000}$

3 a) $125^{\frac{2}{3}} = (\sqrt[3]{125})^2 = 5^2 = \mathbf{25}$

b) $9^{\frac{3}{2}} = (\sqrt{9})^3 = 3^3 = \mathbf{27}$

c) $1000^{\frac{5}{3}} = (\sqrt[3]{1000})^5 = 10^5 = \mathbf{100\,000}$

d) $8000^{\frac{4}{3}} = (\sqrt[3]{8000})^4 = 20^4 = \mathbf{160\,000}$

Model Solutions on Surds

Exercise 1

1 a) $\sqrt{12} = \sqrt{4\times 3} = \sqrt{4} \times \sqrt{3} = \mathbf{2\sqrt{3}}$

b) $\sqrt{20} = \sqrt{4\times 5} = \sqrt{4} \times \sqrt{5} = \mathbf{2\sqrt{5}}$

c) $\sqrt{50} = \sqrt{25\times 2} = \sqrt{25} \times \sqrt{2} = \mathbf{5\sqrt{2}}$

d) $\sqrt{32} = \sqrt{16\times 2} = \sqrt{16} \times \sqrt{2} = \mathbf{4\sqrt{2}}$

e) $\sqrt{108} = \sqrt{36\times 3} = \sqrt{36} \times \sqrt{3} = \mathbf{6\sqrt{3}}$

f) $\sqrt{300} = \sqrt{100\times 3} = \sqrt{100} \times \sqrt{3} = \mathbf{10\sqrt{3}}$

g) $\sqrt{98} = \sqrt{49\times 2} = \sqrt{49} \times \sqrt{2} = \mathbf{7\sqrt{2}}$

h) $\sqrt{192} = \sqrt{64\times 3} = \sqrt{64} \times \sqrt{3} = \mathbf{8\sqrt{3}}$

2 a) $\sqrt{2} \times \sqrt{24} = \sqrt{48} = \sqrt{16\times 3} = \mathbf{4\sqrt{3}}$

b) $\sqrt{3} \times \sqrt{12} = \sqrt{36} = \mathbf{6}$

In this case $6\sqrt{1}$ is simplified to 6 as $\sqrt{1} = 1$.

c) $\sqrt{3} \times \sqrt{24} = \sqrt{72} = \sqrt{36\times 2} = \mathbf{6\sqrt{2}}$

d) $\sqrt{2} \times \sqrt{10} = \sqrt{20} = \sqrt{4\times 5} = \mathbf{2\sqrt{5}}$

e) $\sqrt{40} \times \sqrt{2} = \sqrt{80} = \sqrt{16\times 5} = \mathbf{4\sqrt{5}}$

f) $\sqrt{3} \times \sqrt{60} = \sqrt{180} = \sqrt{36\times 5} = \mathbf{6\sqrt{5}}$

g) $\sqrt{7} \times \sqrt{35} = \sqrt{245} = \sqrt{49\times 5} = \mathbf{7\sqrt{5}}$

h) $\sqrt{50} \times \sqrt{10} = \sqrt{500} = \sqrt{100\times 5} = \mathbf{10\sqrt{5}}$

i) $\sqrt{8} \times \sqrt{24} = \sqrt{192} = \sqrt{64\times 3} = = \mathbf{8\sqrt{3}}$

Exercise 2

1 a) $\sqrt{90} \div \sqrt{10} = \sqrt{90 \div 10} = \sqrt{9} = \mathbf{3}$

b) $\sqrt{72} \div \sqrt{2} = \sqrt{72 \div 2} = \sqrt{36} = \mathbf{6}$

c) $\sqrt{200} \div \sqrt{8} = \sqrt{200 \div 8} = \sqrt{25} = \mathbf{5}$

d) $\sqrt{243} \div \sqrt{3} = \sqrt{243 \div 3} = \sqrt{81} = \mathbf{9}$

e) $\sqrt{294} \div \sqrt{6} = \sqrt{294 \div 6} = \sqrt{49} = \mathbf{7}$

f) $\sqrt{80} \div \sqrt{10} = \sqrt{80 \div 10} = \sqrt{8} = \mathbf{2\sqrt{2}}$

g) $\sqrt{120} \div \sqrt{10} = \sqrt{120 \div 10} = \sqrt{12} = \mathbf{2\sqrt{3}}$

h) $\sqrt{180} \div \sqrt{3} = \sqrt{180 \div 3} = \sqrt{60} = \mathbf{2\sqrt{15}}$

i) $\sqrt{180} \div \sqrt{9} = \sqrt{180 \div 9} = \sqrt{20} = \mathbf{2\sqrt{5}}$

j) $\sqrt{96} \div \sqrt{6} = \sqrt{96 \div 6} = \sqrt{16} = \mathbf{4}$

k) $\sqrt{484} \div \sqrt{22} = \sqrt{484 \div 22} = \mathbf{\sqrt{22}}$

l) $\sqrt{210} \div \sqrt{35} = \sqrt{210 \div 35} = \mathbf{\sqrt{6}}$

2 a) $\sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \mathbf{\frac{1}{3}}$

b) $\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \mathbf{\frac{2}{5}}$

c) $\sqrt{\frac{49}{121}} = \frac{\sqrt{49}}{\sqrt{121}} = \mathbf{\frac{7}{11}}$

d) $\sqrt{\frac{100}{64}} = \frac{\sqrt{100}}{\sqrt{64}} = \frac{10}{8} = \mathbf{\frac{5}{4}}$

e) $\sqrt{\frac{18}{200}} = \frac{\sqrt{18}}{\sqrt{200}} = \frac{3\sqrt{2}}{10\sqrt{2}} = \mathbf{\frac{3}{10}}$

f) $\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \mathbf{\frac{\sqrt{2}}{5}}$

g) $\sqrt{\frac{108}{147}} = \frac{\sqrt{108}}{\sqrt{147}} = \frac{6\sqrt{3}}{7\sqrt{3}} = \mathbf{\frac{6}{7}}$

h) $\sqrt{\frac{27}{64}} = \frac{\sqrt{27}}{\sqrt{64}} = \mathbf{\frac{3\sqrt{3}}{8}}$

i) $\sqrt{\frac{98}{121}} = \frac{\sqrt{98}}{\sqrt{121}} = \mathbf{\frac{7\sqrt{2}}{11}}$

Exercise 3

1 a) $2\sqrt{3} + 3\sqrt{3} = \mathbf{5\sqrt{3}}$

b) $7\sqrt{7} - 3\sqrt{7} = \mathbf{4\sqrt{7}}$

c) $\mathbf{2\sqrt{3} + 3\sqrt{7}}$

Not all sums of surds can be simplified.

d) $2\sqrt{32} + 3\sqrt{2} = 2 \times 4\sqrt{2} + 3\sqrt{2} = \mathbf{11\sqrt{2}}$

e) $2\sqrt{27} - 3\sqrt{3} = 2 \times 3\sqrt{3} - 3\sqrt{3} = \mathbf{3\sqrt{3}}$

f) $5\sqrt{7} + 3\sqrt{28} = 5\sqrt{7} + 3 \times 2\sqrt{7} = \mathbf{11\sqrt{7}}$

2 a) $2\sqrt{125} - 3\sqrt{80} = 2 \times 5\sqrt{5} - 3 \times 4\sqrt{5} = \mathbf{-2\sqrt{5}}$

b) $\sqrt{108} + 2\sqrt{300} = 6\sqrt{3} + 2 \times 10\sqrt{3} = \mathbf{26\sqrt{3}}$

c) $5\sqrt{294} - 3\sqrt{216} = 5 \times 7\sqrt{6} - 3 \times 6\sqrt{6} = \mathbf{17\sqrt{6}}$

Exercise 4

1 **a)** $(2 + \sqrt{3})^2 = 2^2 + 2 \times 2\sqrt{3} + 3 = \mathbf{7 + 4\sqrt{3}}$

b) $(1 + \sqrt{2})(1 - \sqrt{2}) = 1^2 - 2 = \mathbf{-1}$

This is the difference of two squares.

c) $(5 - \sqrt{2})^2 = 5^2 - 2 \times 5\sqrt{2} + 2 = \mathbf{27 - 10\sqrt{2}}$

d) $(3 - 3\sqrt{2})(3 - \sqrt{2}) = 9 - 3\sqrt{2} - 9\sqrt{2} + 6$

$= \mathbf{15 - 12\sqrt{2}}$

e) $(5 + \sqrt{3})(3 + \sqrt{3}) = 15 + 5\sqrt{3} + 3\sqrt{3} + 3$

$= \mathbf{18 + 8\sqrt{3}}$

f) $(7 + 2\sqrt{2})(7 - 2\sqrt{2}) = 7^2 - (2\sqrt{2})^2$

$= 49 - 4 \times 2 = \mathbf{41}$

This is also the difference of two squares — but don't forget you need to square the coefficient in front of the $\sqrt{2}$.

2 **a)** $(2 + \sqrt{6})(4 + \sqrt{3}) = 8 + 2\sqrt{3} + 4\sqrt{6} + \sqrt{18}$

$= \mathbf{8 + 2\sqrt{3} + 4\sqrt{6} + 3\sqrt{2}}$

b) $(4 - \sqrt{7})(5 - \sqrt{2}) = \mathbf{20 - 4\sqrt{2} - 5\sqrt{7} + \sqrt{14}}$

c) $(1 - 2\sqrt{10})(6 - \sqrt{15})$

$= 6 - \sqrt{15} - 12\sqrt{10} + 2\sqrt{150}$

$= 6 - \sqrt{15} - 12\sqrt{10} + 2 \times 5\sqrt{6}$

$= \mathbf{6 - \sqrt{15} - 12\sqrt{10} + 10\sqrt{6}}$

Exercise 5

1 **a)** $\frac{6}{\sqrt{6}} = \frac{6\sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{6\sqrt{6}}{6} = \mathbf{\sqrt{6}}$

b) $\frac{8}{\sqrt{8}} = \frac{8\sqrt{8}}{\sqrt{8} \times \sqrt{8}} = \frac{8\sqrt{8}}{8} = \sqrt{8} = \mathbf{2\sqrt{2}}$

c) $\frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{5\sqrt{5}}{5} = \mathbf{\sqrt{5}}$

d) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \mathbf{\frac{\sqrt{3}}{3}}$

e) $\frac{15}{\sqrt{5}} = \frac{15\sqrt{5}}{5} = \mathbf{3\sqrt{5}}$

f) $\frac{9}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = \mathbf{3\sqrt{3}}$

g) $\frac{7}{\sqrt{12}} = \frac{7\sqrt{12}}{12} = \frac{7 \times 2\sqrt{3}}{12} = \mathbf{\frac{7\sqrt{3}}{6}}$

h) $\frac{12}{\sqrt{1000}} = \frac{12\sqrt{1000}}{1000} = \frac{12 \times 10\sqrt{10}}{1000} = \mathbf{\frac{3\sqrt{10}}{25}}$

2 **a)** $\frac{1}{5\sqrt{5}} = \frac{\sqrt{5}}{5\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5}}{5 \times 5} = \mathbf{\frac{\sqrt{5}}{25}}$

b) $\frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{3\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3 \times 3} = \mathbf{\frac{\sqrt{3}}{9}}$

c) $\frac{3}{4\sqrt{8}} = \frac{3\sqrt{8}}{4 \times 8} = \frac{3 \times 2\sqrt{2}}{32} = \mathbf{\frac{3\sqrt{2}}{16}}$

d) $\frac{3}{2\sqrt{5}} = \frac{3\sqrt{5}}{2 \times 5} = \mathbf{\frac{3\sqrt{5}}{10}}$

e) $\frac{2}{7\sqrt{3}} = \frac{2\sqrt{3}}{7 \times 3} = \mathbf{\frac{2\sqrt{3}}{21}}$

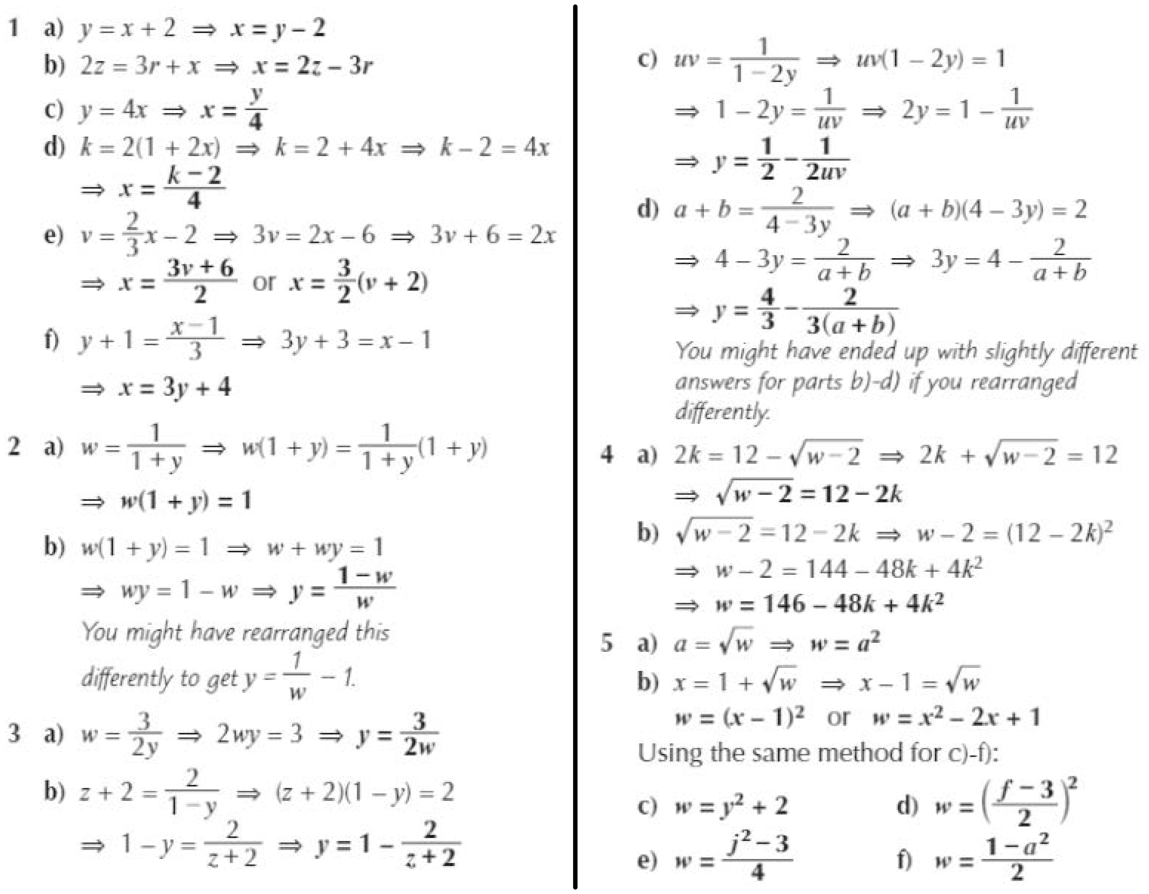
f) $\frac{1}{6\sqrt{12}} = \frac{\sqrt{12}}{6 \times 12} = \frac{2\sqrt{3}}{72} = \mathbf{\frac{\sqrt{3}}{36}}$

g) $\frac{10}{7\sqrt{5}} = \frac{10\sqrt{5}}{7 \times 5} = \frac{10\sqrt{5}}{35} = \mathbf{\frac{2\sqrt{5}}{7}}$

h) $\frac{5}{9\sqrt{10}} = \frac{5\sqrt{10}}{9 \times 10} = \frac{5\sqrt{10}}{90} = \mathbf{\frac{\sqrt{10}}{18}}$

Week 2 answers

Rearranging Formulae



1 a) $y = x + 2 \Rightarrow x = y - 2$

b) $2z = 3r + x \Rightarrow x = 2z - 3r$

c) $y = 4x \Rightarrow x = \frac{y}{4}$

d) $k = 2(1 + 2x) \Rightarrow k = 2 + 4x \Rightarrow k - 2 = 4x$

$\Rightarrow x = \frac{k - 2}{4}$

e) $v = \frac{2}{3}x - 2 \Rightarrow 3v = 2x - 6 \Rightarrow 3v + 6 = 2x$

$\Rightarrow x = \frac{3v + 6}{2}$ or $x = \frac{3}{2}(v + 2)$

f) $y + 1 = \frac{x - 1}{3} \Rightarrow 3y + 3 = x - 1$

$\Rightarrow x = 3y + 4$

2 a) $w = \frac{1}{1 + y} \Rightarrow w(1 + y) = \frac{1}{1 + y}(1 + y)$

$\Rightarrow w(1 + y) = 1$

b) $w(1 + y) = 1 \Rightarrow w + wy = 1$

$\Rightarrow wy = 1 - w \Rightarrow y = \frac{1 - w}{w}$

You might have rearranged this differently to get $y = \frac{1}{w} - 1$.

3 a) $w = \frac{3}{2y} \Rightarrow 2wy = 3 \Rightarrow y = \frac{3}{2w}$

b) $z + 2 = \frac{2}{1 - y} \Rightarrow (z + 2)(1 - y) = 2$

$\Rightarrow 1 - y = \frac{2}{z + 2} \Rightarrow y = 1 - \frac{2}{z + 2}$

c) $uv = \frac{1}{1 - 2y} \Rightarrow uv(1 - 2y) = 1$

$\Rightarrow 1 - 2y = \frac{1}{uv} \Rightarrow 2y = 1 - \frac{1}{uv}$

$\Rightarrow y = \frac{1}{2} - \frac{1}{2uv}$

d) $a + b = \frac{2}{4 - 3y} \Rightarrow (a + b)(4 - 3y) = 2$

$\Rightarrow 4 - 3y = \frac{2}{a + b} \Rightarrow 3y = 4 - \frac{2}{a + b}$

$\Rightarrow y = \frac{4}{3} - \frac{2}{3(a + b)}$

You might have ended up with slightly different answers for parts b)-d) if you rearranged differently.

4 a) $2k = 12 - \sqrt{w - 2} \Rightarrow 2k + \sqrt{w - 2} = 12$

$\Rightarrow \sqrt{w - 2} = 12 - 2k$

b) $\sqrt{w - 2} = 12 - 2k \Rightarrow w - 2 = (12 - 2k)^2$

$\Rightarrow w - 2 = 144 - 48k + 4k^2$

$\Rightarrow w = 146 - 48k + 4k^2$

5 a) $a = \sqrt{w} \Rightarrow w = a^2$

b) $x = 1 + \sqrt{w} \Rightarrow x - 1 = \sqrt{w}$

$w = (x - 1)^2$ or $w = x^2 - 2x + 1$

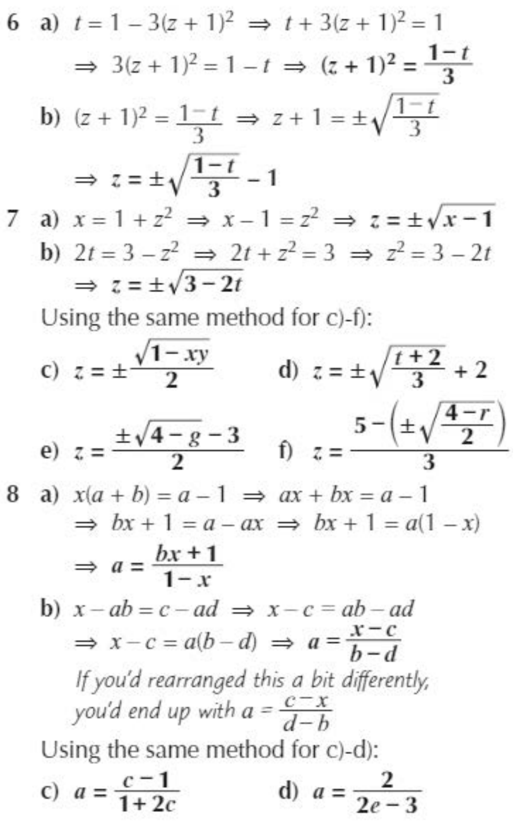
Using the same method for c)-f):

c) $w = y^2 + 2$

d) $w = \left(\frac{f - 3}{2}\right)^2$

e) $w = \frac{j^2 - 3}{4}$

f) $w = \frac{1 - a^2}{2}$



6 **a)** $t = 1 - 3(z+1)^2 \Rightarrow t + 3(z+1)^2 = 1$

$\Rightarrow 3(z+1)^2 = 1 - t \Rightarrow \mathbf{(z+1)^2 = \frac{1-t}{3}}$

b) $(z+1)^2 = \frac{1-t}{3} \Rightarrow z + 1 = \pm\sqrt{\frac{1-t}{3}}$

$\Rightarrow \mathbf{z = \pm\sqrt{\frac{1-t}{3}} - 1}$

7 **a)** $x = 1 + z^2 \Rightarrow x - 1 = z^2 \Rightarrow \mathbf{z = \pm\sqrt{x-1}}$

b) $2t = 3 - z^2 \Rightarrow 2t + z^2 = 3 \Rightarrow z^2 = 3 - 2t$

$\Rightarrow \mathbf{z = \pm\sqrt{3-2t}}$

Using the same method for c)-f):

c) $\mathbf{z = \pm\frac{\sqrt{1-xy}}{2}}$ **d)** $\mathbf{z = \pm\sqrt{\frac{t+2}{3}} + 2}$

e) $\mathbf{z = \frac{\pm\sqrt{4-g}-3}{2}}$ **f)** $\mathbf{z = \frac{5-\left(\pm\sqrt{\frac{4-r}{2}}\right)}{3}}$

8 **a)** $x(a+b) = a - 1 \Rightarrow ax + bx = a - 1$

$\Rightarrow bx + 1 = a - ax \Rightarrow bx + 1 = a(1-x)$

$\Rightarrow \mathbf{a = \frac{bx+1}{1-x}}$

b) $x - ab = c - ad \Rightarrow x - c = ab - ad$

$\Rightarrow x - c = a(b-d) \Rightarrow \mathbf{a = \frac{x-c}{b-d}}$

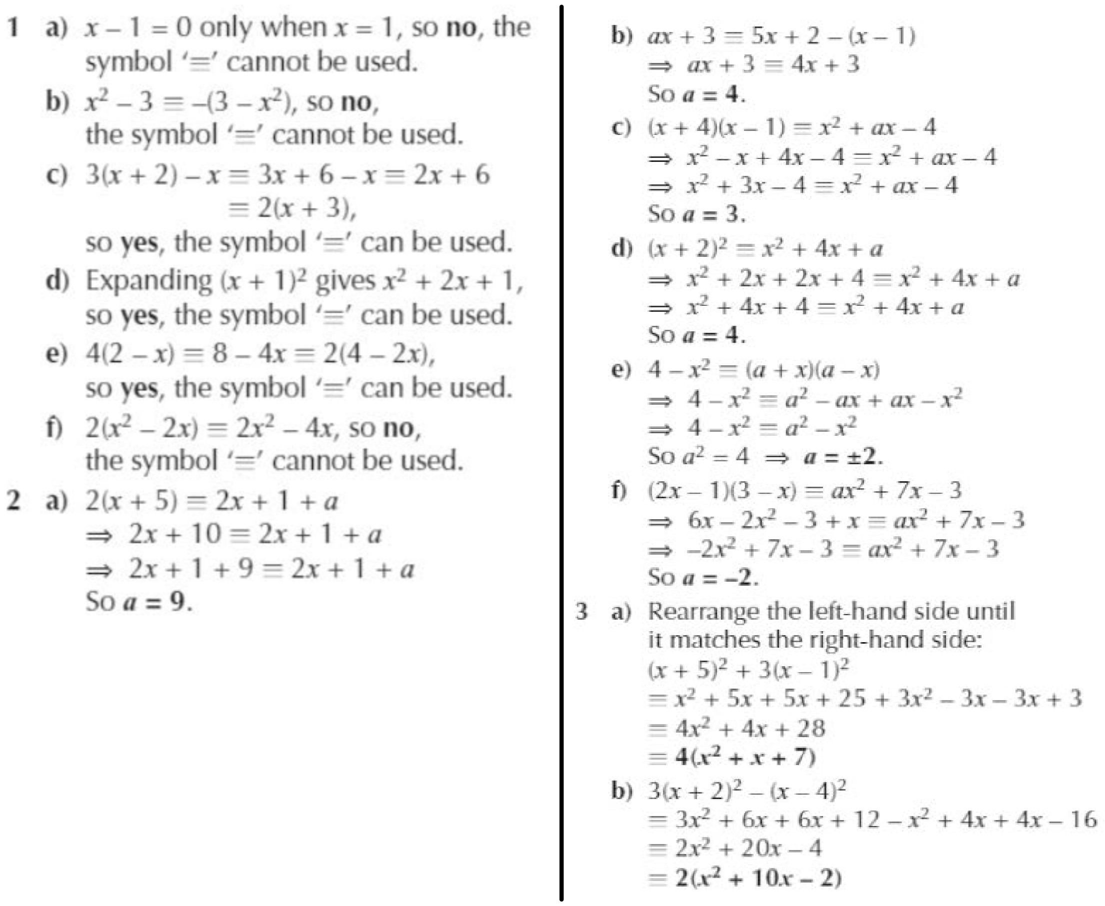
If you'd rearranged this a bit differently, you'd end up with $a = \frac{c-x}{d-b}$

Using the same method for c)-d):

c) $\mathbf{a = \frac{c-1}{1+2c}}$ **d)** $\mathbf{a = \frac{2}{2e-3}}$

Model Solutions on

Identities



1 **a)** $x - 1 = 0$ only when $x = 1$, so **no**, the symbol '≡' cannot be used.

b) $x^2 - 3 \equiv -(3 - x^2)$, so **no**, the symbol '≡' cannot be used.

c) $3(x + 2) - x \equiv 3x + 6 - x \equiv 2x + 6$
$\equiv 2(x + 3)$,
so **yes**, the symbol '≡' can be used.

d) Expanding $(x + 1)^2$ gives $x^2 + 2x + 1$, so **yes**, the symbol '≡' can be used.

e) $4(2 - x) \equiv 8 - 4x \equiv 2(4 - 2x)$, so **yes**, the symbol '≡' can be used.

f) $2(x^2 - 2x) \equiv 2x^2 - 4x$, so **no**, the symbol '≡' cannot be used.

2 **a)** $2(x + 5) \equiv 2x + 1 + a$
$\Rightarrow\ 2x + 10 \equiv 2x + 1 + a$
$\Rightarrow\ 2x + 1 + 9 \equiv 2x + 1 + a$
So $\boldsymbol{a = 9}$.

b) $ax + 3 \equiv 5x + 2 - (x - 1)$
$\Rightarrow\ ax + 3 \equiv 4x + 3$
So $\boldsymbol{a = 4}$.

c) $(x + 4)(x - 1) \equiv x^2 + ax - 4$
$\Rightarrow\ x^2 - x + 4x - 4 \equiv x^2 + ax - 4$
$\Rightarrow\ x^2 + 3x - 4 \equiv x^2 + ax - 4$
So $\boldsymbol{a = 3}$.

d) $(x + 2)^2 \equiv x^2 + 4x + a$
$\Rightarrow\ x^2 + 2x + 2x + 4 \equiv x^2 + 4x + a$
$\Rightarrow\ x^2 + 4x + 4 \equiv x^2 + 4x + a$
So $\boldsymbol{a = 4}$.

e) $4 - x^2 \equiv (a + x)(a - x)$
$\Rightarrow\ 4 - x^2 \equiv a^2 - ax + ax - x^2$
$\Rightarrow\ 4 - x^2 \equiv a^2 - x^2$
So $a^2 = 4 \Rightarrow \boldsymbol{a = \pm 2}$.

f) $(2x - 1)(3 - x) \equiv ax^2 + 7x - 3$
$\Rightarrow\ 6x - 2x^2 - 3 + x \equiv ax^2 + 7x - 3$
$\Rightarrow\ -2x^2 + 7x - 3 \equiv ax^2 + 7x - 3$
So $\boldsymbol{a = -2}$.

3 **a)** Rearrange the left-hand side until it matches the right-hand side:
$(x + 5)^2 + 3(x - 1)^2$
$\equiv x^2 + 5x + 5x + 25 + 3x^2 - 3x - 3x + 3$
$\equiv 4x^2 + 4x + 28$
$\equiv \mathbf{4(x^2 + x + 7)}$

b) $3(x + 2)^2 - (x - 4)^2$
$\equiv 3x^2 + 6x + 6x + 12 - x^2 + 4x + 4x - 16$
$\equiv 2x^2 + 20x - 4$
$\equiv \mathbf{2(x^2 + 10x - 2)}$

Model Solutions on

Solving Quadratic Equations by Factorising

Exercise 1

1 a) $x(x+8)=0 \Rightarrow x=0$ or $x+8=0$
$\Rightarrow x=0$ or $x=-8$

b) $(x-5)(x-1)=0 \Rightarrow x-5=0$ or $x-1=0$
$\Rightarrow x=5$ or $x=1$

c) $(x+2)(x+6)=0 \Rightarrow x+2=0$ or $x+6=0$
$\Rightarrow x=-2$ or $x=-6$

d) $(x-9)(x+7)=0 \Rightarrow x-9=0$ or $x+7=0$
$\Rightarrow x=9$ or $x=-7$

2 a) $x(x-3)=0 \Rightarrow x=0$ or $x=3$

b) $x(x+12)=0 \Rightarrow x=0$ or $x=-12$

c) $(x+2)(x+1)=0 \Rightarrow x=-2$ or $x=-1$

d) $(x-1)^2=0 \Rightarrow x=1$

e) $(x+2)^2=0 \Rightarrow x=-2$

f) $(x+4)(x-1)=0 \Rightarrow x=-4$ or $x=1$

g) $(x-4)(x+1)=0 \Rightarrow x=4$ or $x=-1$

h) $(x+4)(x+1) \Rightarrow x=-4$ or $x=-1$

i) $(x-3)(x-2)=0 \Rightarrow x=3$ or $x=2$

j) $(x+6)(x+2)=0 \Rightarrow x=-6$ or $x=-2$

k) $(x-6)(x+4)=0 \Rightarrow x=6$ or $x=-4$

l) $(x-12)(x-3)=0 \Rightarrow x=12$ or $x=3$

3 a) $x(2x-3)=0 \Rightarrow x=0$ or $2x-3=0$
$\Rightarrow x=0$ or $2x=3$
$\Rightarrow x=0$ or $x=\frac{3}{2}$

b) $(x-2)(3x-1)=0$
$\Rightarrow x-2=0$ or $3x-1=0$
$\Rightarrow x=2$ or $3x=1$
$\Rightarrow x=2$ or $x=\frac{1}{3}$

c) $(3x+4)(2x+5)=0$
$\Rightarrow 3x+4=0$ or $2x+5=0$
$\Rightarrow 3x=-4$ or $2x=-5$
$\Rightarrow x=-\frac{4}{3}$ or $x=-\frac{5}{2}$

d) $(4x-7)(5x+2)=0$
$\Rightarrow 4x-7=0$ or $5x+2=0$
$\Rightarrow 4x=7$ or $5x=-2$
$\Rightarrow x=\frac{7}{4}$ or $x=-\frac{2}{5}$

4 a) $x(3x+5)=0 \Rightarrow x=0$ or $x=-\frac{5}{3}$

b) $(2x+3)(x-1)=0 \Rightarrow x=-\frac{3}{2}$ or $x=1$

c) $(5x-2)(x+1)=0 \Rightarrow x=\frac{2}{5}$ or $x=-1$

d) $(3x-2)(x-3)=0 \Rightarrow x=\frac{2}{3}$ or $x=3$

e) $(4x+1)(x+4)=0 \Rightarrow x=-\frac{1}{4}$ or $x=-4$

f) $(6x-11)(x+2)=0 \Rightarrow x=\frac{11}{6}$ or $x=-2$

g) $(2x-5)^2=0 \Rightarrow x=\frac{5}{2}$

h) $(3x-2)^2=0 \Rightarrow x=\frac{2}{3}$

Exercise 2

1 a) $x^2=x \Rightarrow x^2-x=0 \Rightarrow x(x-1)=0$
$\Rightarrow x=0$ or $x=1$

b) $x^2+2x=3 \Rightarrow x^2+2x-3=0$
$\Rightarrow (x+3)(x-1)=0 \Rightarrow x=-3$ or $x=1$

Using the same method for c)-l):

c) $(x-7)(x-3)=0 \Rightarrow x=7$ or $x=3$

d) $(x-4)(x-2)=0 \Rightarrow x=4$ or $x=2$

e) $(x-6)(x-2)=0 \Rightarrow x=6$ or $x=2$

f) $(3x-9)(x+1)=0 \Rightarrow x=3$ or $x=-1$

g) $(2x-1)(x+11)=0 \Rightarrow x=\frac{1}{2}$ or $x=-11$

h) $(2x+3)(2x-1)=0 \Rightarrow x=-\frac{3}{2}$ or $x=\frac{1}{2}$

i) $(3x-1)(2x+1)=0 \Rightarrow x=\frac{1}{3}$ or $x=-\frac{1}{2}$

j) $(3x-2)(2x-1)=0 \Rightarrow x=\frac{2}{3}$ or $x=\frac{1}{2}$

k) $(2x-1)^2=0 \Rightarrow x=\frac{1}{2}$

l) $(3x-5)^2=0 \Rightarrow x=\frac{5}{3}$

2 a) $x(x-2)=8 \Rightarrow x^2-2x=8$
$\Rightarrow x^2-2x-8=0$
$\Rightarrow (x-4)(x+2)=0 \Rightarrow x=4$ or $x=-2$

b) $x(x+2)=35 \Rightarrow x^2+2x=35$
$\Rightarrow x^2+2x-35=0$
$\Rightarrow (x+7)(x-5)=0 \Rightarrow x=-7$ or $x=5$

Using the same method for c)-i):

c) $(x+6)^2=0 \Rightarrow x=-6$

d) $(x-7)^2=0 \Rightarrow x=7$

e) $(x+2)(x-2)=0 \Rightarrow x=-2$ or $x=2$
This one's the difference of two squares (see p.84).

f) $(2x+1)(x+7)=0 \Rightarrow x=-\frac{1}{2}$ or $x=-7$

f) $(2x+1)(x+7)=0 \Rightarrow x=-\frac{1}{2}$ or $x=-7$

g) $(3x+4)(3x-3)=0 \Rightarrow x=-\frac{4}{3}$ or $x=1$

h) $(3x-1)(x-3)=0 \Rightarrow x=\frac{1}{3}$ or $x=3$

i) $(4x-1)(2x-1)=0 \Rightarrow x=\frac{1}{4}$ or $x=\frac{1}{2}$

3 a) $x+1=\frac{6}{x} \Rightarrow x(x+1)=6 \Rightarrow x^2+x=6$
$\Rightarrow x^2+x-6=0$
$\Rightarrow (x+3)(x-2)=0 \Rightarrow x=-3$ or $x=2$

b) $x-2=\frac{4}{x+1} \Rightarrow (x-2)(x+1)=4$
$\Rightarrow x^2+x-2x-2=4$
$\Rightarrow x^2-x-6=0$
$\Rightarrow (x+2)(x-3)=0 \Rightarrow x=-2$ or $x=3$

Using the same method for c)-f):

c) $(x+6)(x-5)=0 \Rightarrow x=-6$ or $x=5$

d) $(2x-3)(x-2)=0 \Rightarrow x=\frac{3}{2}$ or $x=2$

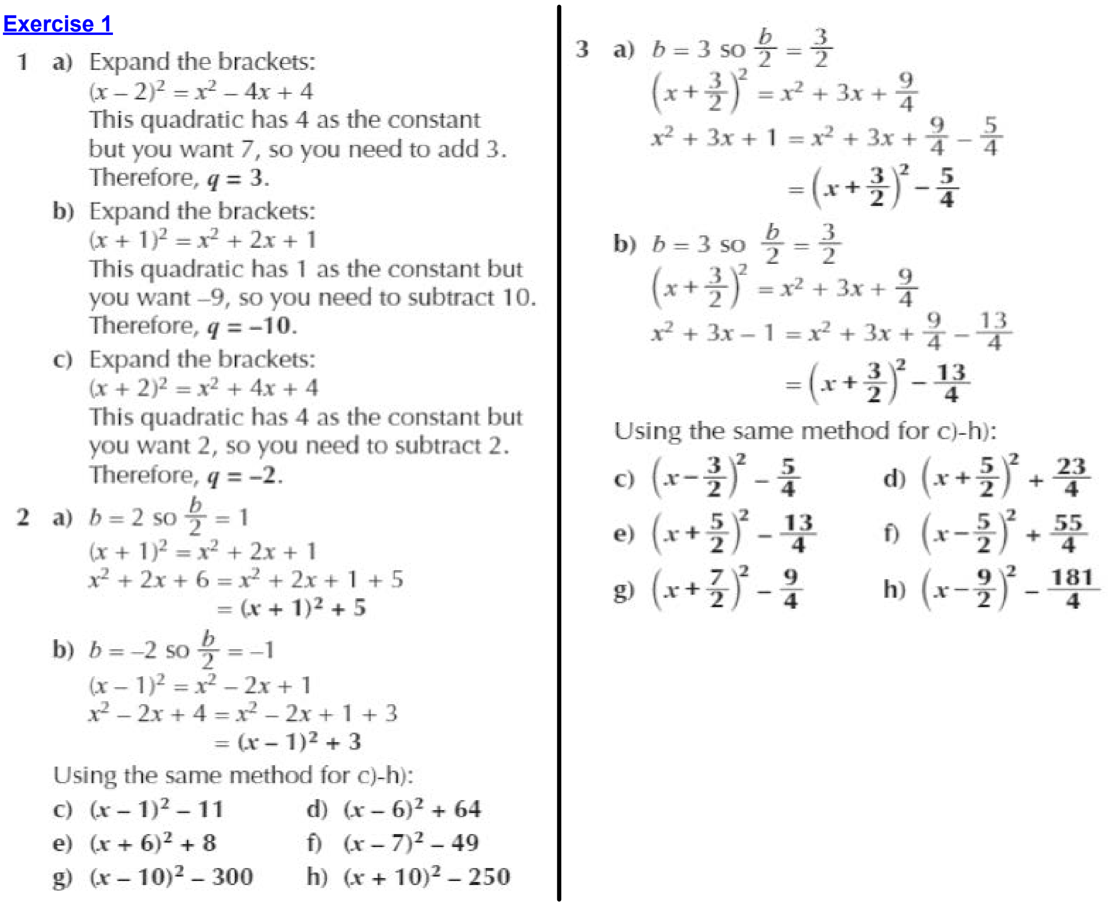
e) $(6x-5)(x+1) \Rightarrow x=\frac{5}{6}$ or $x=-1$

f) $(12x-5)(2x+1)=0 \Rightarrow x=\frac{5}{12}$ or $x=-\frac{1}{2}$

Model Solutions on

Completing the Square

Exercise 1



1 a) Expand the brackets:
$(x - 2)^2 = x^2 - 4x + 4$
This quadratic has 4 as the constant but you want 7, so you need to add 3.
Therefore, $q = 3$.

b) Expand the brackets:
$(x + 1)^2 = x^2 + 2x + 1$
This quadratic has 1 as the constant but you want –9, so you need to subtract 10.
Therefore, $q = -10$.

c) Expand the brackets:
$(x + 2)^2 = x^2 + 4x + 4$
This quadratic has 4 as the constant but you want 2, so you need to subtract 2.
Therefore, $q = -2$.

2 a) $b = 2$ so $\frac{b}{2} = 1$
$(x + 1)^2 = x^2 + 2x + 1$
$x^2 + 2x + 6 = x^2 + 2x + 1 + 5$
$= (x + 1)^2 + 5$

b) $b = -2$ so $\frac{b}{2} = -1$
$(x - 1)^2 = x^2 - 2x + 1$
$x^2 - 2x + 4 = x^2 - 2x + 1 + 3$
$= (x - 1)^2 + 3$

Using the same method for c)-h):

c) $(x - 1)^2 - 11$
d) $(x - 6)^2 + 64$
e) $(x + 6)^2 + 8$
f) $(x - 7)^2 - 49$
g) $(x - 10)^2 - 300$
h) $(x + 10)^2 - 250$

3 a) $b = 3$ so $\frac{b}{2} = \frac{3}{2}$
$\left(x + \frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4}$
$x^2 + 3x + 1 = x^2 + 3x + \frac{9}{4} - \frac{5}{4}$
$= \left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$

b) $b = 3$ so $\frac{b}{2} = \frac{3}{2}$
$\left(x + \frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4}$
$x^2 + 3x - 1 = x^2 + 3x + \frac{9}{4} - \frac{13}{4}$
$= \left(x + \frac{3}{2}\right)^2 - \frac{13}{4}$

Using the same method for c)-h):

c) $\left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$
d) $\left(x + \frac{5}{2}\right)^2 + \frac{23}{4}$
e) $\left(x + \frac{5}{2}\right)^2 - \frac{13}{4}$
f) $\left(x - \frac{5}{2}\right)^2 + \frac{55}{4}$
g) $\left(x + \frac{7}{2}\right)^2 - \frac{9}{4}$
h) $\left(x - \frac{9}{2}\right)^2 - \frac{181}{4}$

Exercise 2

1 a) Expand the brackets:

$2\left(x+\frac{1}{4}\right)^2 = 2x^2 + x + \frac{1}{8}$

This quadratic has $\frac{1}{8}$ as the constant

but you want 0, so you need to

subtract $\frac{1}{8}$. Therefore, $\boldsymbol{q = -\frac{1}{8}}$.

b) Expand the brackets:

$2(x+5)^2 = 2x^2 + 20x + 50$

This quadratic has 50 as the constant but you want 500, so you need to add 450. Therefore, $\boldsymbol{q = 450}$.

c) Expand the brackets:

$3\left(x+\frac{2}{3}\right)^2 = 3x^2 + 4x + \frac{4}{3}$

This quadratic has $\frac{4}{3}$ as the constant but you want 25, so you need to add $23\frac{2}{3} = \frac{71}{3}$. Therefore, $\boldsymbol{q = \frac{71}{3}}$.

d) Expand the brackets:

$4\left(x-\frac{7}{8}\right)^2 = 4x^2 - 7x + \frac{49}{16}$

This quadratic has $\frac{49}{16}$ as the constant but you want –1, so you need to subtract $1\frac{49}{16} = \frac{65}{16}$. Therefore, $\boldsymbol{q = -\frac{65}{16}}$.

2 a) $p = \frac{b}{2a} = \frac{-12}{2 \times 2} = \mathbf{-3}$

To find q, expand the brackets:

$2(x-3)^2 = 2x^2 - 12x + 18$

This quadratic has 18 as the constant but you want 9, so you need to subtract 9. Therefore, $\boldsymbol{q = -9}$.

b) $p = \frac{b}{2a} = \frac{-5}{2 \times 3} = \boldsymbol{-\frac{5}{6}}$

To find q, expand the brackets:

$3\left(x-\frac{5}{6}\right)^2 = 3x^2 - 5x + \frac{25}{12}$

This quadratic has $\frac{25}{12} = 2\frac{1}{12}$ as the constant but you want –1, so you need to subtract $3\frac{1}{12} = \frac{37}{12}$. Therefore $\boldsymbol{q = -\frac{37}{12}}$.

3 a) $a\left(x+\frac{b}{2a}\right)^2 = 2\left(x+\frac{8}{4}\right)^2 = 2(x+2)^2$

$= 2(x^2 + 4x + 4) = 2x^2 + 8x + 8$

$2x^2 + 8x + 81 = 2x^2 + 8x + 8 + 73$

$= \mathbf{2(x+2)^2 + 73}$

b) $a\left(x+\frac{b}{2a}\right)^2 = 3\left(x+\frac{8}{6}\right)^2 = 3\left(x+\frac{4}{3}\right)^2$

$= 3\left(x^2 + \frac{8}{3}x + \frac{16}{9}\right)$

$= 3x^2 + 8x + \frac{16}{3}$

$3x^2 + 8x + 10 = 3x^2 + 8x + \frac{16}{3} + \frac{14}{3}$

$= \boldsymbol{3\left(x+\frac{4}{3}\right)^2 + \frac{14}{3}}$

Using the same method for c)-i):

c) $\boldsymbol{2\left(x-\frac{1}{2}\right)^2 + \frac{5}{2}}$ d) $\boldsymbol{5\left(x+\frac{1}{2}\right)^2 - \frac{9}{4}}$

e) $\boldsymbol{2\left(x-\frac{1}{4}\right)^2 + \frac{7}{8}}$ f) $\mathbf{3(x+3)^2 + 63}$

g) $\boldsymbol{4\left(x+\frac{13}{8}\right)^2 - \frac{25}{16}}$ h) $\boldsymbol{2\left(x+\frac{11}{4}\right)^2 - \frac{121}{8}}$

i) $\boldsymbol{-2\left(x-\frac{5}{2}\right)^2 + \frac{21}{2}}$

Here, $a = -2$ so the bracket is

$-2\left(x+\frac{10}{2 \times -2}\right)^2 = -2\left(x-\frac{5}{2}\right)^2.$

Exercise 3

1 a) $(x-1)^2 = x^2 - 2x + 1$

$x^2 - 2x - 4 = x^2 - 2x + 1 - 5 = (x-1)^2 - 5$

$x^2 - 2x - 4 = 0 \Rightarrow (x-1)^2 - 5 = 0$

$\Rightarrow (x-1)^2 = 5 \Rightarrow x - 1 = \pm\sqrt{5}$

$\Rightarrow \boldsymbol{x = 1 \pm \sqrt{5}}$

b) $(x+2)^2 = x^2 + 4x + 4$

$x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x+2)^2 - 1$

$x^2 + 4x + 3 = 0 \Rightarrow (x+2)^2 - 1 = 0$

$\Rightarrow (x+2)^2 = 1 \Rightarrow x + 2 = \pm 1$

$\Rightarrow x = -2 \pm 1 \Rightarrow \boldsymbol{x = -1}$ or $\boldsymbol{x = -3}$

This quadratic factorises quite easily but the question tells you to complete the square.

Using the same method for c)-f):

c) $\boldsymbol{x = -3 \pm \sqrt{13}}$ d) $\boldsymbol{x = -4 \pm 2\sqrt{3}}$

e) $\boldsymbol{x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}}$ f) $\boldsymbol{x = \frac{11}{2} \pm \frac{\sqrt{21}}{2}}$

2 a) $(x+3)^2 = x^2 + 6x + 9$

$x^2 + 6x + 4 = x^2 + 6x + 9 - 5 = (x+3)^2 - 5$

$x^2 + 6x = 4 = 0 \Rightarrow (x+3)^2 - 5 = 0$

$\Rightarrow (x+3)^2 = 5 \Rightarrow x + 3 = \pm\sqrt{5}$

$\Rightarrow x = -3 \pm \sqrt{5}$

$\Rightarrow x = -0.7639... = \mathbf{-0.76}$ (2 d.p.)

or $x = -5.2360... = \mathbf{-5.24}$ (2 d.p.)

b) $(x-1)^2 = x^2 - 2x + 1$

$x^2 - 2x - 5 = x^2 - 2x + 1 - 6 = (x-1)^2 - 6$

$x^2 - 2x - 5 = 0 \Rightarrow (x-1)^2 - 6 = 0$

$\Rightarrow (x-1)^2 = 6 \Rightarrow x - 1 = \pm\sqrt{6}$

$\Rightarrow x = 1 \pm \sqrt{6}$

$\Rightarrow x = 3.4494... = \mathbf{3.45}$ (2 d.p.)

or $x = -1.4494... = \mathbf{-1.45}$ (2 d.p.)

Using the same method for c)-f):

c) $x = 0.4641... = \mathbf{0.46}$ (2 d.p.)

or $x = -6.4641... = \mathbf{-6.46}$ (2 d.p.)

d) $x = -1.1715... = \mathbf{-1.17}$ (2 d.p.)

or $x = -6.8284... = \mathbf{-6.83}$ (2 d.p.)

e) $x = 3.7015... = \mathbf{3.70}$ (2 d.p.)

or $x = -2.7015... = \mathbf{-2.70}$ (2 d.p.)

f) $x = 4.3027... = \mathbf{4.30}$ (2 d.p.)

or $x = 0.6972... = \mathbf{0.70}$ (2 d.p.)

3 a) $a\left(x+\frac{b}{2a}\right)^2 = 3\left(x+\frac{2}{6}\right)^2 = 3\left(x+\frac{1}{3}\right)^2$

$= 3\left(x^2 + \frac{2}{3}x + \frac{1}{9}\right)$

$= 3x^2 + 2x + \frac{1}{3}$

$3x^2 + 2x - 2 = 3x^2 + 2x + \frac{1}{3} - \frac{7}{3}$

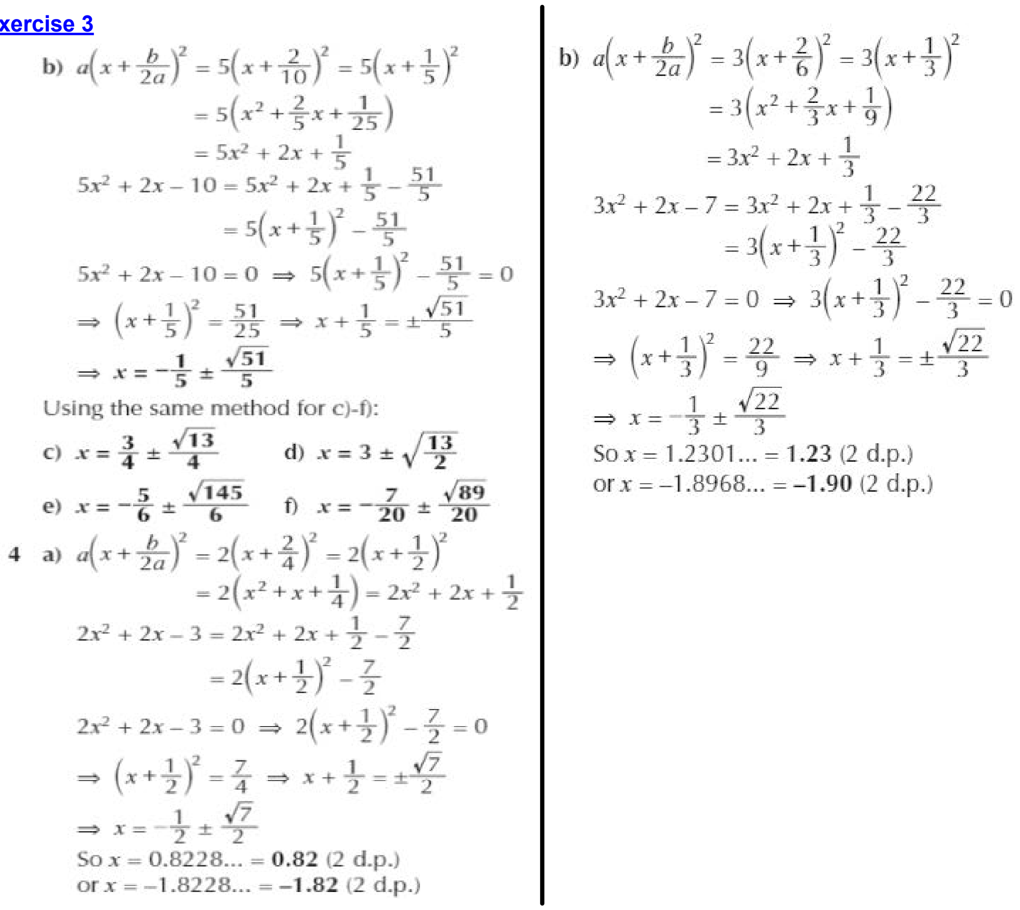
$= 3\left(x+\frac{1}{3}\right)^2 - \frac{7}{3}$

$3x^2 + 2x - 2 = 0 \Rightarrow 3\left(x+\frac{1}{3}\right)^2 - \frac{7}{3} = 0$

$\Rightarrow \left(x+\frac{1}{3}\right)^2 = \frac{7}{9} \Rightarrow x + \frac{1}{3} = \pm\frac{\sqrt{7}}{3}$

$\Rightarrow \boldsymbol{x = -\frac{1}{3} \pm \frac{\sqrt{7}}{3}}$

Exercise 3



b) $a\left(x+\frac{b}{2a}\right)^2 = 5\left(x+\frac{2}{10}\right)^2 = 5\left(x+\frac{1}{5}\right)^2$

$= 5\left(x^2+\frac{2}{5}x+\frac{1}{25}\right)$

$= 5x^2 + 2x + \frac{1}{5}$

$5x^2 + 2x - 10 = 5x^2 + 2x + \frac{1}{5} - \frac{51}{5}$

$= 5\left(x+\frac{1}{5}\right)^2 - \frac{51}{5}$

$5x^2 + 2x - 10 = 0 \Rightarrow 5\left(x+\frac{1}{5}\right)^2 - \frac{51}{5} = 0$

$\Rightarrow \left(x+\frac{1}{5}\right)^2 = \frac{51}{25} \Rightarrow x + \frac{1}{5} = \pm\frac{\sqrt{51}}{5}$

$\Rightarrow \boldsymbol{x = -\frac{1}{5} \pm \frac{\sqrt{51}}{5}}$

Using the same method for c)-f):

c) $\boldsymbol{x = \frac{3}{4} \pm \frac{\sqrt{13}}{4}}$ d) $\boldsymbol{x = 3 \pm \sqrt{\frac{13}{2}}}$

e) $\boldsymbol{x = -\frac{5}{6} \pm \frac{\sqrt{145}}{6}}$ f) $\boldsymbol{x = -\frac{7}{20} \pm \frac{\sqrt{89}}{20}}$

4 a) $a\left(x+\frac{b}{2a}\right)^2 = 2\left(x+\frac{2}{4}\right)^2 = 2\left(x+\frac{1}{2}\right)^2$

$= 2\left(x^2+x+\frac{1}{4}\right) = 2x^2 + 2x + \frac{1}{2}$

$2x^2 + 2x - 3 = 2x^2 + 2x + \frac{1}{2} - \frac{7}{2}$

$= 2\left(x+\frac{1}{2}\right)^2 - \frac{7}{2}$

$2x^2 + 2x - 3 = 0 \Rightarrow 2\left(x+\frac{1}{2}\right)^2 - \frac{7}{2} = 0$

$\Rightarrow \left(x+\frac{1}{2}\right)^2 = \frac{7}{4} \Rightarrow x + \frac{1}{2} = \pm\frac{\sqrt{7}}{2}$

$\Rightarrow x = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}$

So $x = 0.8228\ldots = \mathbf{0.82}$ (2 d.p.)
or $x = -1.8228\ldots = \mathbf{-1.82}$ (2 d.p.)

b) $a\left(x+\frac{b}{2a}\right)^2 = 3\left(x+\frac{2}{6}\right)^2 = 3\left(x+\frac{1}{3}\right)^2$

$= 3\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)$

$= 3x^2 + 2x + \frac{1}{3}$

$3x^2 + 2x - 7 = 3x^2 + 2x + \frac{1}{3} - \frac{22}{3}$

$= 3\left(x+\frac{1}{3}\right)^2 - \frac{22}{3}$

$3x^2 + 2x - 7 = 0 \Rightarrow 3\left(x+\frac{1}{3}\right)^2 - \frac{22}{3} = 0$

$\Rightarrow \left(x+\frac{1}{3}\right)^2 = \frac{22}{9} \Rightarrow x + \frac{1}{3} = \pm\frac{\sqrt{22}}{3}$

$\Rightarrow x = -\frac{1}{3} \pm \frac{\sqrt{22}}{3}$

So $x = 1.2301\ldots = \mathbf{1.23}$ (2 d.p.)
or $x = -1.8968\ldots = \mathbf{-1.90}$ (2 d.p.)

Model Solutions on

The Quadratic Formula

Exercise 1

1 **a)** $a = 1,\ b = -3,\ c = 1$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{9-4}}{2} = \mathbf{\frac{3 \pm \sqrt{5}}{2}}$$

b) $a = 1,\ b = -2,\ c = -12$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-12)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{4+48}}{2} = \frac{2 \pm \sqrt{52}}{2}$$

But $\sqrt{52} = \sqrt{4 \times 13} = 2\sqrt{13}$ and so

$$x = \frac{2 \pm 2\sqrt{13}}{2} = \mathbf{1 \pm \sqrt{13}}$$

Using the same method for c)-i):

c) $a = 4,\ b = -3,\ c = -8$
Then $x = \mathbf{\frac{3 \pm \sqrt{137}}{8}}$

d) $a = 1,\ b = 1,\ c = -1$
Then $x = \mathbf{\frac{-1 \pm \sqrt{5}}{2}}$

e) $a = 1,\ b = -8,\ c = -5$
Then $x = \mathbf{4 \pm \sqrt{21}}$

f) $a = 3,\ b = 6,\ c = -5$
Then $x = \mathbf{\frac{-3 \pm 2\sqrt{6}}{3}}$

g) $a = 1,\ b = -5,\ c = -3$
Then $x = \mathbf{\frac{5 \pm \sqrt{37}}{2}}$

h) $a = -2,\ b = 8,\ c = 13$
Then $x = \mathbf{\frac{4 \pm \sqrt{42}}{2}}$

i) $a = 1,\ b = -7,\ c = 3$
Then $x = \mathbf{\frac{7 \pm \sqrt{37}}{2}}$

2 **a)** $a = 1,\ b = 3,\ c = 1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$$

So $x = -0.3819... = \mathbf{-0.38}$ (2 d.p.)
or $x = -2.6180... = \mathbf{-2.62}$ (2 d.p.)

b) $a = 3,\ b = 2,\ c = -2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times (-2)}}{2 \times 3}$$

$$= \frac{-2 \pm \sqrt{4+24}}{6} = \frac{-2 \pm \sqrt{28}}{6}$$

So $x = 0.5485... = \mathbf{0.55}$ (2 d.p.)
or $x = -1.2152... = \mathbf{-1.22}$ (2 d.p.)

Using the same method for c)-i):

c) $a = 1,\ b = -3,\ c = -3$
Then $x = 3.7912... = \mathbf{3.79}$ (2 d.p.)
or $x = -0.7912... = \mathbf{-0.79}$ (2 d.p.)

d) $a = 1,\ b = 5,\ c = -4$
Then $x = 0.7015... = \mathbf{0.70}$ (2 d.p.)
or $x = -5.7015... = \mathbf{-5.70}$ (2 d.p.)

e) $a = -1,\ b = -8,\ c = 11$
Then $x = 1.1961... = \mathbf{1.20}$ (2 d.p.)
or $x = -9.1961... = \mathbf{-9.20}$ (2 d.p.)

f) $a = 1,\ b = -7,\ c = -6$
Then $x = 7.7720... = \mathbf{7.77}$ (2 d.p.)
or $x = -0.7720... = \mathbf{-0.77}$ (2 d.p.)

Exercise 1

g) $a = 1,\ b = 6,\ c = -2$
Then $x = 0.3166... = \mathbf{0.32}$ (2 d.p.)
or $x = -6.3166... = \mathbf{-6.32}$ (2 d.p.)

h) $a = 1,\ b = 4,\ c = -1$
Then $x = 0.2360... = \mathbf{0.24}$ (2 d.p.)
or $x = -4.2360... = \mathbf{-4.24}$ (2 d.p.)

i) $a = 2,\ b = 8,\ c = 3$
Then $x = -0.4188... = \mathbf{-0.42}$ (2 d.p.)
or $x = -3.5811... = \mathbf{-3.58}$ (2 d.p.)

3 **a)** $x^2 + 3x = 6 \Rightarrow x^2 + 3x - 6 = 0$
$a = 1,\ b = 3,\ c = -6$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-6)}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{9+24}}{2} = \frac{-3 \pm \sqrt{33}}{2}$$

So $x = \mathbf{\frac{-3 \pm \sqrt{33}}{2}}$

b) $x^2 - 5x + 11 = 2x + 3 \Rightarrow x^2 - 7x + 8 = 0$
$a = 1,\ b = -7,\ c = 8$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$= \frac{7 \pm \sqrt{49-32}}{2} = \frac{7 \pm \sqrt{17}}{2}$$

So $x = \mathbf{\frac{7 \pm \sqrt{17}}{2}}$

Using the same method for c)-i):

c) $a = 1,\ b = -7,\ c = 7$
Then $x = \mathbf{\frac{7 \pm \sqrt{21}}{2}}$

d) $a = 1,\ b = 2,\ c = -11$
Then $x = \mathbf{-1 + 2\sqrt{3}}$ or $x = \mathbf{-1 - 2\sqrt{3}}$

e) $a = 1,\ b = 3,\ c = -5$
Then $x = \mathbf{\frac{-3 \pm \sqrt{29}}{2}}$

f) $a = 1,\ b = -3,\ c = -9$
Then $x = \mathbf{\frac{3 \pm 3\sqrt{5}}{2}}$

g) $a = 1,\ b = -6,\ c = 4$
Then $x = \mathbf{3 \pm \sqrt{5}}$

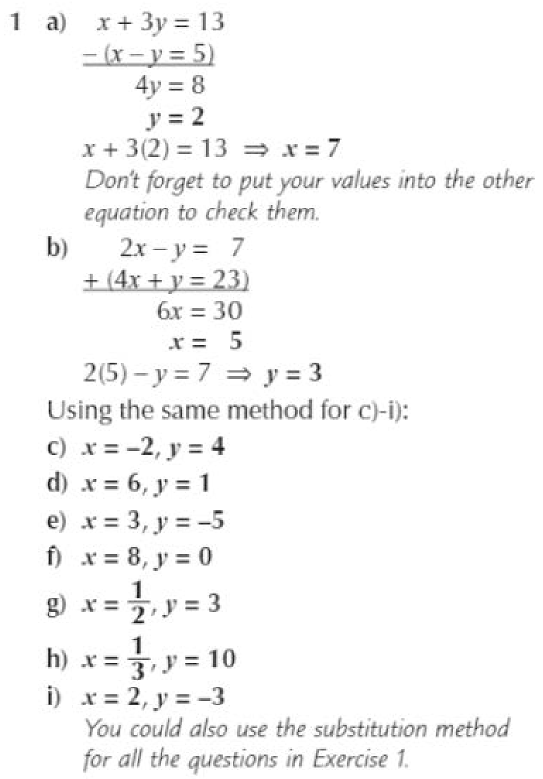
h) $a = 1,\ b = 2,\ c = -6$
Then $x = \mathbf{-1 \pm \sqrt{7}}$

i) $a = 2,\ b = -3,\ c = -7$
Then $x = \mathbf{\frac{3 \pm \sqrt{65}}{4}}$

Model Solutions on

Simultaneous Linear Equations

Exercise 1



1 a) $x + 3y = 13$

$\underline{-(x - y = 5)}$

$4y = 8$

$\mathbf{y = 2}$

$x + 3(2) = 13 \Rightarrow \mathbf{x = 7}$

Don't forget to put your values into the other equation to check them.

b) $2x - y = 7$

$\underline{+(4x + y = 23)}$

$6x = 30$

$\mathbf{x = 5}$

$2(5) - y = 7 \Rightarrow \mathbf{y = 3}$

Using the same method for c)-i):

c) $\mathbf{x = -2, y = 4}$

d) $\mathbf{x = 6, y = 1}$

e) $\mathbf{x = 3, y = -5}$

f) $\mathbf{x = 8, y = 0}$

g) $\mathbf{x = \frac{1}{2}, y = 3}$

h) $\mathbf{x = \frac{1}{3}, y = 10}$

i) $\mathbf{x = 2, y = -3}$

You could also use the substitution method for all the questions in Exercise 1.

Exercise 2

1 **a)** (1) $3x + 2y = 16$, (2) $2x + y = 9$
(1) $3x + 2y = 16$
(2) × 2: $\underline{-(4x + 2y = 18)}$
$-x = -2$
$\mathbf{x = 2}$
$2 \times 2 + y = 9 \Rightarrow \mathbf{y = 5}$

b) (1) $4x + 3y = 16$, (2) $5x - y = 1$
(1) $4x + 3y = 16$
(2) × 3: $\underline{+(15x - 3y = 3)}$
$19x = 19$
$\mathbf{x = 1}$
$5 \times 1 - y = 1 \Rightarrow \mathbf{y = 4}$

Using the same method for c)-f):

c) $\mathbf{x = 6, y = 2}$
d) $\mathbf{x = 5, y = 0}$
e) $\mathbf{x = 3, y = 2}$
f) $\mathbf{e = -1, r = -4}$

2 **a)** (1) $3x - 2y = 8$, (2) $5x - 3y = 14$
(2) × 2: $(10x - 6y = 28)$
(1) × 3: $\underline{-(9x - 6y = 24)}$
$x = \mathbf{4}$
$3(4) - 2y = 8 \Rightarrow 2y = 4 \Rightarrow \mathbf{y = 2}$

b) (1) $4p + 3q = 17$, (2) $3p - 4q = 19$
(1) × 4: $(16p + 12q = 68)$
(2) × 3: $\underline{+(9p - 12q = 57)}$
$25p = 125$
$\mathbf{p = 5}$
$4(5) + 3q = 17 \Rightarrow 3q = -3 \Rightarrow \mathbf{q = -1}$

Using the same method for c)-f):

c) $\mathbf{u = 2, v = 1}$
d) $\mathbf{c = 8, d = \frac{1}{2}}$
e) $\mathbf{r = -2, s = 4}$
f) $\mathbf{m = 3, n = 1}$

Exercise 3

1 $x + y = 58$
$\underline{+(x - y = 22)}$
$2x = 80$
$\mathbf{x = 40}$
$40 + y = 58 \Rightarrow \mathbf{y = 18}$

2 Let d = sherbet dip, c = chocolate bar
(1) $4d + 3c = 1.91$, (2) $3d + 4c = 1.73$
(1) × 4: $(16d + 12c = 7.64)$
(2) × 3: $\underline{-(9d + 12c = 5.19)}$
$7d = 2.45$
$d = 0.35$
$4 \times 0.35 + 3c = 1.91 \Rightarrow 3c = 0.51 \Rightarrow c = 0.17$
So a sherbet dip costs **35p** and a chocolate bar costs **17p**

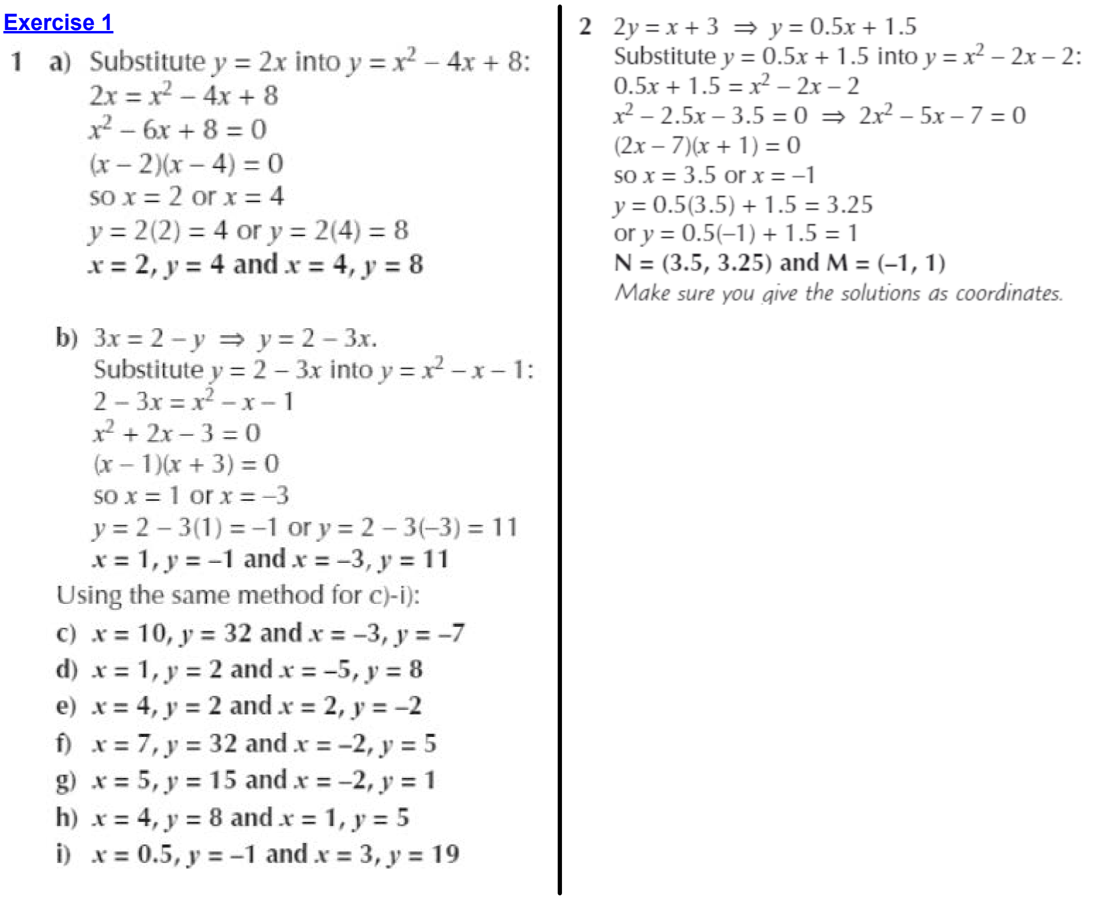
3 Let y = yellow aliens, b = blue spiders
(1) $7y + 5b = 85$, (2) $6y + 11b = 93$
(1) × 6: $(42y + 30b = 510)$
(2) × 7: $\underline{-(42y + 77b = 651)}$
$-47b = -141$
$b = 3$
$7y + 5(3) = 85 \Rightarrow 7y = 70 \Rightarrow y = 10$
So Hal's score = $8(10) + 3$ = **83 points**.

4 $3(x + y) = 5x + 2y - 1$
$\Rightarrow 3x + 3y - 5x - 2y = -1$
$\Rightarrow 2x - y = 1$ (1)
$3(x + y) = 4x + 4 + y$
$\Rightarrow 3x + 3y = 4x + 4 + y$
$\Rightarrow x - 2y = -4$ (2)
(1) $2x - y = 1$
$\underline{(2) \times 2: -(2x - 4y = -8)}$
$3y = 9$
$y = 3$
$2x - 3 = 1 \Rightarrow 2x = 4, x = 2$
$3(x + y) = 3(2 + 3) =$ **15 cm**
Your working might look a bit different if you used different pairs of sides.

Model Solutions on

Simultaneous Linear and Quadratic Equations

Exercise 1



1 **a)** Substitute $y = 2x$ into $y = x^2 - 4x + 8$:
$2x = x^2 - 4x + 8$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
so $x = 2$ or $x = 4$
$y = 2(2) = 4$ or $y = 2(4) = 8$
$x = 2, y = 4$ and $x = 4, y = 8$

b) $3x = 2 - y \Rightarrow y = 2 - 3x$.
Substitute $y = 2 - 3x$ into $y = x^2 - x - 1$:
$2 - 3x = x^2 - x - 1$
$x^2 + 2x - 3 = 0$
$(x - 1)(x + 3) = 0$
so $x = 1$ or $x = -3$
$y = 2 - 3(1) = -1$ or $y = 2 - 3(-3) = 11$
$x = 1, y = -1$ and $x = -3, y = 11$

Using the same method for c)-i):

c) **$x = 10, y = 32$ and $x = -3, y = -7$**
d) **$x = 1, y = 2$ and $x = -5, y = 8$**
e) **$x = 4, y = 2$ and $x = 2, y = -2$**
f) **$x = 7, y = 32$ and $x = -2, y = 5$**
g) **$x = 5, y = 15$ and $x = -2, y = 1$**
h) **$x = 4, y = 8$ and $x = 1, y = 5$**
i) **$x = 0.5, y = -1$ and $x = 3, y = 19$**

2 $2y = x + 3 \Rightarrow y = 0.5x + 1.5$
Substitute $y = 0.5x + 1.5$ into $y = x^2 - 2x - 2$:
$0.5x + 1.5 = x^2 - 2x - 2$
$x^2 - 2.5x - 3.5 = 0 \Rightarrow 2x^2 - 5x - 7 = 0$
$(2x - 7)(x + 1) = 0$
so $x = 3.5$ or $x = -1$
$y = 0.5(3.5) + 1.5 = 3.25$
or $y = 0.5(-1) + 1.5 = 1$
N = (3.5, 3.25) and M = (–1, 1)
Make sure you give the solutions as coordinates.

Exercise 1

3 Substitute $y = 4 - 3x$ into $y = 6x^2 + 10x - 1$:
$4 - 3x = 6x^2 + 10x - 1$
$6x^2 + 13x - 5 = 0$
$(3x - 1)(2x + 5) = 0$
so $x = \frac{1}{3}$ or $-\frac{5}{2}$
$y = 4 - 3\left(\frac{1}{3}\right) = 3$ or $y = 4 - 3(-\frac{5}{2}) = \frac{23}{2}$
$(\frac{1}{3}, 3)$ and $(-\frac{5}{2}, \frac{23}{2})$

4 $x^2 + 3x + 1 = 5x$
$x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
so **$x = 1$** and **$y = 5$**
The line and curve only meet at one point, so the line is a tangent to the curve.

5 $x - 4y = 2 \Rightarrow x = 2 + 4y$
$y^2 + y(2 + 4y) = 0$
$5y^2 + 2y = 0 \Rightarrow y(5y + 2) = 0$
so $y = 0$ or $y = -0.4$
$x = 2 + 4(0) = 2$ or $x = 2 + 4(-0.4) = 0.4$
$x = 2, y = 0$ and $x = 0.4, y = -0.4$

6 a) $x + y = 7 \Rightarrow y = 7 - x$
$x^2 - x(7 - x) - 4 = 0$
$2x^2 - 7x - 4 = 0$
$(2x + 1)(x - 4) = 0$
so $x = -0.5$ or $x = 4$
$y = 7 - (-0.5) = 7.5$ or $y = 7 - 4 = 3$
$x = -0.5, y = 7.5$ and $x = 4, y = 3$

b) $x + y = 5 \Rightarrow x = 5 - y$
$5 - y + y(5 - y) + 2y^2 = 2$
$2y^2 - y^2 + 5y - y + 5 - 2 = 0$
$y^2 + 4y + 3 = 0$
$(y + 1)(y + 3) = 0$
so $y = -1$ or $y = -3$
$x = 5 - (-1) = 6$ or $x = 5 - (-3) = 8$
$x = 6, y = -1$ and $x = 8, y = -3$

Using the same method for c)-f):

c) **$x = 1, y = 1$ and $x = 4, y = -2$**

d) **$x = 2, y = -2$ and $x = -3, y = -7$**

e) **$x = 2, y = 2$ and $x = 4, y = 0$**

f) **$x = 16, y = -1.5$ and $x = -2, y = 3$**

Exercise 2

1 a) $2x + y = 3 \Rightarrow y = 3 - 2x$
$(3 - 2x)^2 - x^2 = 0$
$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
so $x = 1$ or $x = 3$
$y = 3 - 2(1) = 1$ or $y = 3 - 2(3) = -3$
$x = 1, y = 1$ and $x = 3, y = -3$

b) $3x + y = 4 \Rightarrow y = 4 - 3x$
$x^2 + 3x(4 - 3x) + (4 - 3x)^2 = -16$
$x^2 + 12x - 9x^2 + 16 - 24x + 9x^2 + 16 = 0$
$x^2 - 12x + 32 = 0$
$(x - 8)(x - 4) = 0$
so $x = 8$ or $x = 4$
$y = 4 - 3(8) = -20$ or $y = 4 - 3(4) = -8$
$x = 8, y = -20$ and $x = 4, y = -8$

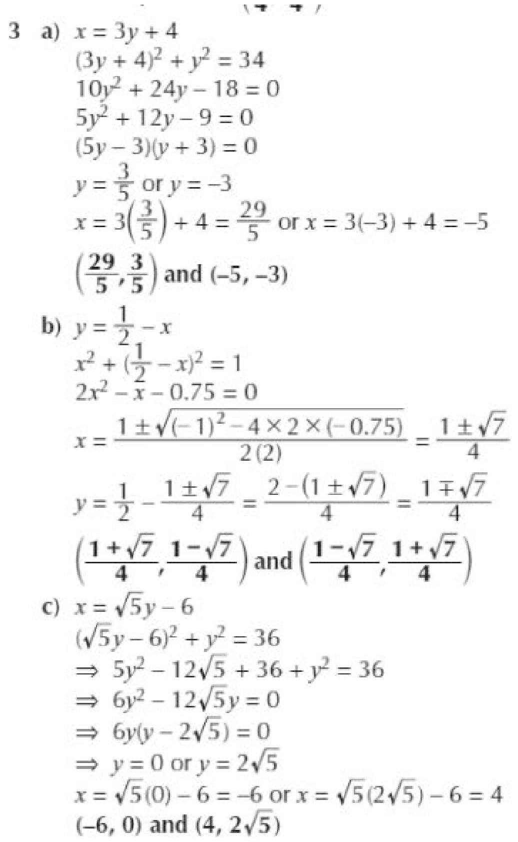
c) $x - y = -4 \Rightarrow y = x + 4$
$x^2 + (x + 4)^2 - x = 20$
$x^2 + x^2 + 8x + 16 - x - 20 = 0$
$2x^2 + 7x - 4 = 0$
$(2x - 1)(x + 4) = 0$
so $x = \frac{1}{2}$ or $x = -4$
$y = \frac{1}{2} + 4 = \frac{9}{2}$ or $y = (-4) + 4 = 0$
$x = \frac{1}{2}, y = \frac{9}{2}$ and $x = -4, y = 0$

2 a) $x - y = -3 \Rightarrow y = x + 3$
$3x^2 + 7x + (x + 3)^2 = 21$
$3x^2 + 7x + x^2 + 6x + 9 = 21$
$4x^2 + 13x - 12 = 0$

b) $(4x - 3)(x + 4) = 0$
so **$x = \frac{3}{4}$** or **$x = -4$**

c) $y = \frac{3}{4} + 3 = \frac{15}{4}$ or $y = (-4) + 3 = -1$
So they cross at **$\left(\frac{3}{4}, \frac{15}{4}\right)$ and $(-4, -1)$**

Exercise 2



3 **a)** $x = 3y + 4$

$(3y + 4)^2 + y^2 = 34$

$10y^2 + 24y - 18 = 0$

$5y^2 + 12y - 9 = 0$

$(5y - 3)(y + 3) = 0$

$y = \frac{3}{5}$ or $y = -3$

$x = 3\left(\frac{3}{5}\right) + 4 = \frac{29}{5}$ or $x = 3(-3) + 4 = -5$

$\left(\mathbf{\frac{29}{5}}, \mathbf{\frac{3}{5}}\right)$ **and (–5, –3)**

b) $y = \frac{1}{2} - x$

$x^2 + (\frac{1}{2} - x)^2 = 1$

$2x^2 - x - 0.75 = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times (-0.75)}}{2(2)} = \frac{1 \pm \sqrt{7}}{4}$

$y = \frac{1}{2} - \frac{1 \pm \sqrt{7}}{4} = \frac{2 - (1 \pm \sqrt{7})}{4} = \frac{1 \mp \sqrt{7}}{4}$

$\left(\mathbf{\frac{1 + \sqrt{7}}{4}}, \mathbf{\frac{1 - \sqrt{7}}{4}}\right)$ **and** $\left(\mathbf{\frac{1 - \sqrt{7}}{4}}, \mathbf{\frac{1 + \sqrt{7}}{4}}\right)$

c) $x = \sqrt{5}y - 6$

$(\sqrt{5}y - 6)^2 + y^2 = 36$

$\Rightarrow 5y^2 - 12\sqrt{5} + 36 + y^2 = 36$

$\Rightarrow 6y^2 - 12\sqrt{5}y = 0$

$\Rightarrow 6y(y - 2\sqrt{5}) = 0$

$\Rightarrow y = 0$ or $y = 2\sqrt{5}$

$x = \sqrt{5}(0) - 6 = -6$ or $x = \sqrt{5}(2\sqrt{5}) - 6 = 4$

(–6, 0) and (4, $\mathbf{2\sqrt{5}}$**)**

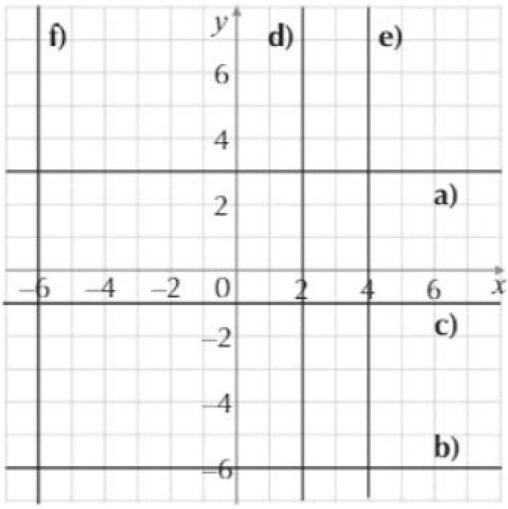
Week 3 answers

1. Horizontal & Vertical Lines

Exercise 1

1 All points on line A have an x-coordinate of –5, so the equation is $x = -5$.
All points on line B have an x-coordinate of –2, so the equation is $x = -2$.
All points on line C have a y-coordinate of 2, so the equation is $y = 2$.
All points on line D have an x-coordinate of 5, so the equation is $x = 5$.
All points on line E have a y-coordinate of –3, so the equation is $y = -3$.

2



3 a) $y = 8$ b) $x = -2$
c) $x = 1$ d) $y = 6$

4 a) (8, –11) b) (–5, –13)

c) $(-\frac{6}{11}, -500)$

Model solutions on

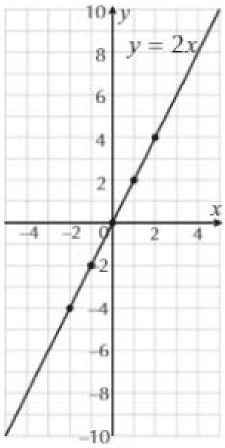
2. Straight Line Graphs

Exercise 2

1 a)

x	–2	–1	0	1	2
y	**–4**	**–2**	**0**	**2**	**4**
Coord.	**(–2, –4)**	**(–1, –2)**	**(0, 0)**	**(1, 2)**	**(2, 4)**

b)-d)

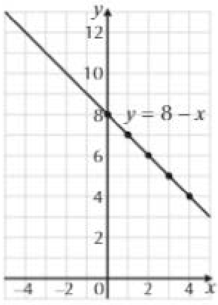


e) (i) **8** (ii) **–6** (iii) **–5**

2 a)

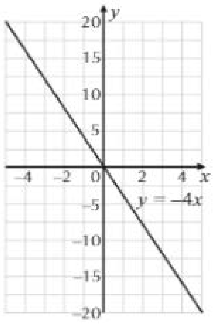
x	0	1	2	3	4
y	**8**	**7**	**6**	**5**	**4**
Coord.	**(0, 8)**	**(1, 7)**	**(2, 6)**	**(3, 5)**	**(4, 4)**

b)-c)

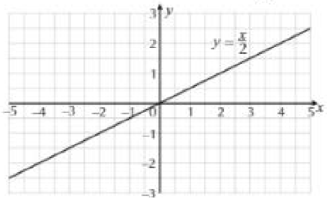


Exercise 2

3 **a)** Find the coordinates of at least two points on the line: e.g. when $x = -5$, $y = -4(-5) = 20$ and when $x = 0$, $y = -4(0) = 0$. Plot these points, join with a straight line, then extend the line to cover the whole range of x.

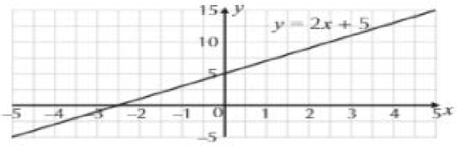


b) Find the coordinates of at least two points on the line: e.g. when $x = -2$, $y = \frac{-2}{2} = -1$ and when $x = 4$, $y = \frac{4}{2} = 2$. Plot these points, join with a straight line, then extend the line to cover the whole range of x.

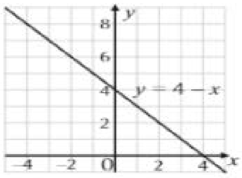


Using the same method for c)-f) and Q4:

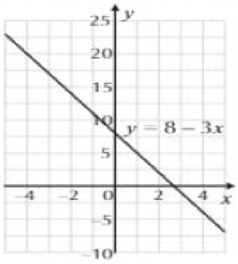
c)



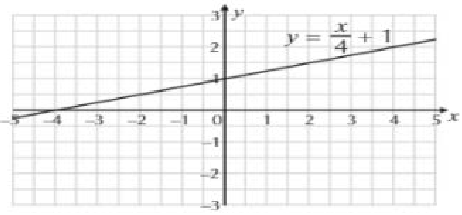
d)



e)

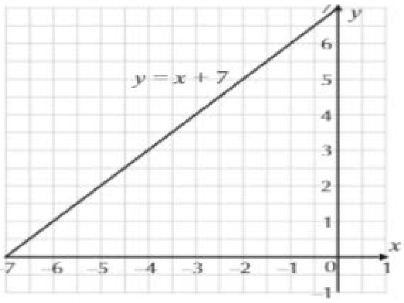


f)

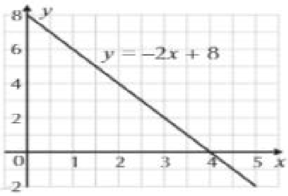


Exercise 2

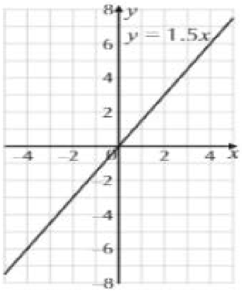
4 **a)**



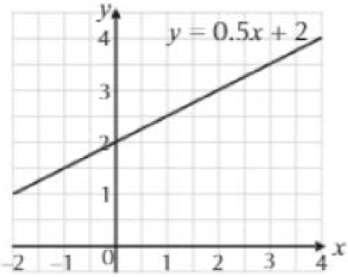
b)



c)



d)



Model solutions on

3. Gradients

Exercise 1

1 a) The line slopes downwards from left to right, so the gradient is **negative**.

b) $y_2 - y_1 = 2 - 6 = \mathbf{-4}$

c) $x_2 - x_1 = 6 - 1 = \mathbf{5}$

d) gradient $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{-4}{5} = \mathbf{-\dfrac{4}{5}}$

2 a) gradient $= \dfrac{3-5}{6-1} = \mathbf{-\dfrac{2}{5}}$

b) gradient $= \dfrac{1-6}{6-(-4)} = \dfrac{-5}{10} = \mathbf{-\dfrac{1}{2}}$

c) gradient $= \dfrac{6-0}{4-(-5)} = \dfrac{6}{9} = \mathbf{\dfrac{2}{3}}$

3 a) (i) $G = \mathbf{(2, -5)}$, $H = \mathbf{(6, 6)}$

(ii) gradient $= \dfrac{6-(-5)}{6-2} = \dfrac{11}{4} = \mathbf{2.75}$

b) (i) $I = \mathbf{(-10, 5)}$, $J = \mathbf{(30, -25)}$

(ii) gradient $= \dfrac{(-25)-5}{30-(-10)} = \dfrac{-30}{40} = \mathbf{-\dfrac{3}{4}}$

c) (i) $K = \mathbf{(-8, -25)}$, $L = \mathbf{(8, 35)}$
$M = \mathbf{(-4, 30)}$, $N = \mathbf{(6, -15)}$

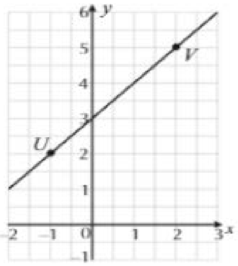
(ii) Line 1 gradient $= \dfrac{35-(-25)}{8-(-8)}$

$= \dfrac{60}{16} = \dfrac{15}{4} = \mathbf{3.75}$

Line 2 gradient $= \dfrac{(-15)-30}{6-(-4)}$

$= \dfrac{-45}{10} = -\dfrac{9}{2} = \mathbf{-4.5}$

4 a)



b) gradient $= \dfrac{5-2}{2-(-1)} = \dfrac{3}{3} = \mathbf{1}$

5 a) $0 - (-3) = \mathbf{3}$ b) $2 - (-4) = \mathbf{6}$

c) gradient $= \dfrac{3}{6} = \mathbf{\dfrac{1}{2}}$

6 a) gradient $= \dfrac{10-4}{2-0} = \dfrac{6}{2} = \mathbf{3}$

b) gradient $= \dfrac{11-3}{5-1} = \dfrac{8}{4} = \mathbf{2}$

Using the same method for c)-f):

c) $\mathbf{\dfrac{2}{3}}$ d) $\mathbf{-\dfrac{3}{4}}$ e) $\mathbf{-\dfrac{3}{4}}$ f) $\mathbf{\dfrac{3}{4}}$

7 Pick 2 points on line A, e.g. (0, 0) and (1, 5)
Then gradient $= \dfrac{5-0}{1-0} = \dfrac{5}{1} = \mathbf{5}$

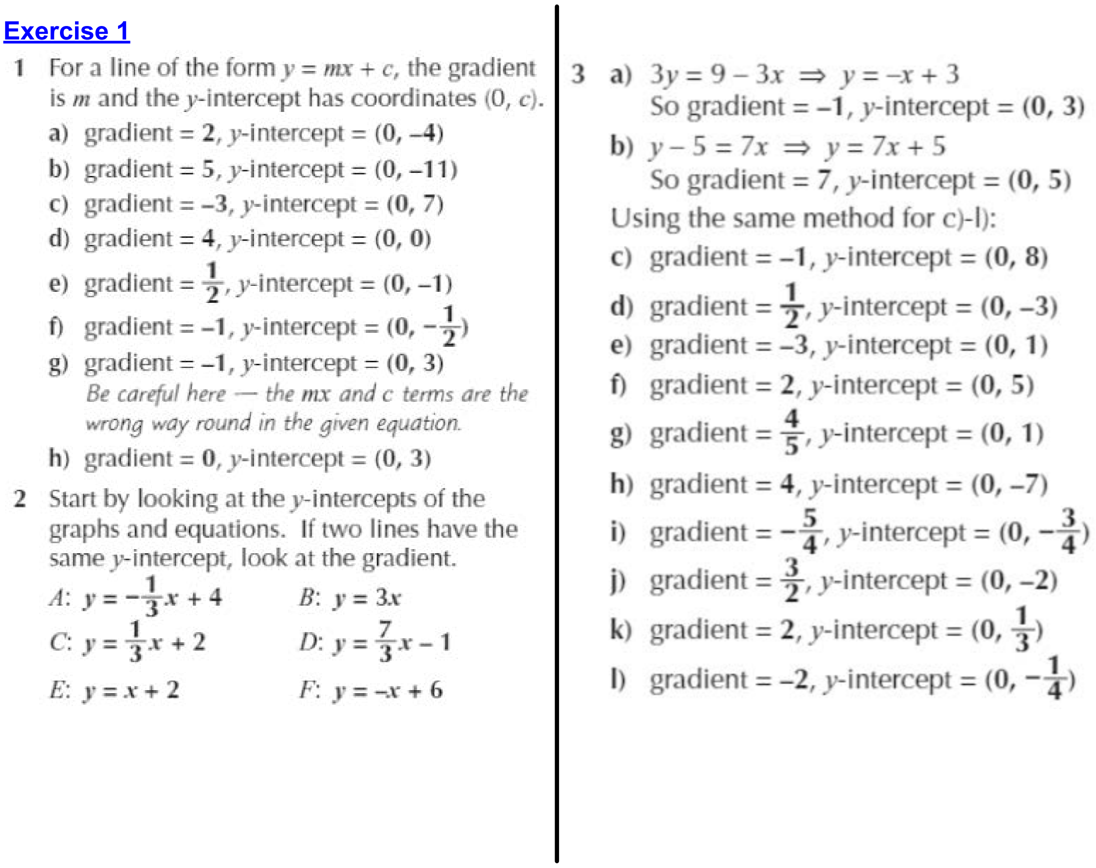
Pick 2 points on line B, e.g. (0, 0) and (4, 1)
Then gradient $= \dfrac{1-0}{4-0} = \mathbf{\dfrac{1}{4}}$

Using the same method for lines C-D:
gradient of $C = \mathbf{-\dfrac{2}{3}}$, gradient of $D = \mathbf{\dfrac{1}{20}}$

Model solutions on

4. Equations of Linear Graphs

Exercise 1



1 For a line of the form $y = mx + c$, the gradient is m and the y-intercept has coordinates $(0, c)$.

- a) gradient = **2**, y-intercept = **(0, –4)**
- b) gradient = **5**, y-intercept = **(0, –11)**
- c) gradient = **–3**, y-intercept = **(0, 7)**
- d) gradient = **4**, y-intercept = **(0, 0)**
- e) gradient = $\frac{1}{2}$, y-intercept = **(0, –1)**
- f) gradient = **–1**, y-intercept = $(0, -\frac{1}{2})$
- g) gradient = **–1**, y-intercept = **(0, 3)**
  Be careful here — the mx and c terms are the wrong way round in the given equation.
- h) gradient = **0**, y-intercept = **(0, 3)**

2 Start by looking at the y-intercepts of the graphs and equations. If two lines have the same y-intercept, look at the gradient.

A: $y = -\frac{1}{3}x + 4$ B: $y = 3x$

C: $y = \frac{1}{3}x + 2$ D: $y = \frac{7}{3}x - 1$

E: $y = x + 2$ F: $y = -x + 6$

3 a) $3y = 9 - 3x \Rightarrow y = -x + 3$
So gradient = **–1**, y-intercept = **(0, 3)**

b) $y - 5 = 7x \Rightarrow y = 7x + 5$
So gradient = **7**, y-intercept = **(0, 5)**

Using the same method for c)-l):

- c) gradient = **–1**, y-intercept = **(0, 8)**
- d) gradient = $\frac{1}{2}$, y-intercept = **(0, –3)**
- e) gradient = **–3**, y-intercept = **(0, 1)**
- f) gradient = **2**, y-intercept = **(0, 5)**
- g) gradient = $\frac{4}{5}$, y-intercept = **(0, 1)**
- h) gradient = **4**, y-intercept = **(0, –7)**
- i) gradient = $-\frac{5}{4}$, y-intercept = $(0, -\frac{3}{4})$
- j) gradient = $\frac{3}{2}$, y-intercept = **(0, –2)**
- k) gradient = **2**, y-intercept = $(0, \frac{1}{3})$
- l) gradient = **–2**, y-intercept = $(0, -\frac{1}{4})$

Exercise 2

1 For parts a) and b) you're given the gradient (m) and the y-intercept (in the form $(0, c)$), so put these values into $y = mx + c$:

a) **$y = 8x + 2$** b) **$y = -x + 7$**

For the rest of the questions, use the gradient and the given point to find the value of c:

c) Gradient = 3, so $10 = 3(1) + c \Rightarrow c = 7$.
So the equation of the line is **$y = 3x + 7$**.

d) Gradient = $\frac{1}{2}$, so $-5 = \frac{1}{2}(4) + c \Rightarrow c = -7$.
So the equation of the line is **$y = \frac{1}{2}x - 7$**.

e) Gradient = -7, so $-4 = -7(2) + c \Rightarrow c = 10$.
So the equation of the line is **$y = -7x + 10$**.

f) Gradient = 5, so $-7 = 5(-3) + c \Rightarrow c = 8$.
So the equation of the line is **$y = 5x + 8$**.

2 a) Gradient = $\frac{11-7}{5-3} = \frac{4}{2} = 2$, so $7 = 2(3) + c$
$\Rightarrow c = 1$. So the equation of the line is
$y = 2x + 1$.

b) Gradient = $\frac{(-5)-1}{2-5} = \frac{-6}{-3} = 2$,
so $1 = 2(5) + c \Rightarrow c = -9$.
So the equation of the line is **$y = 2x - 9$**.

Using the same method for c)-i):

c) **$y = x - 3$** d) **$y = 2x + 5$**
e) **$y = 3x + 2$** f) **$y = 3x + 11$**
g) **$y = -x + 1$** h) **$y = -4x + 7$**
i) **$y = \frac{3}{2}x - \frac{1}{2}$**

3 Gradient of line $A = \frac{4-0}{(-2)-2} = \frac{4}{-4} = -1$,
y-intercept = 2, so its equation is **$y = -x + 2$**.

Gradient of line $B = \frac{5-0}{1-0} = \frac{5}{1} = 5$.
It goes through the origin so y-intercept = 0, so its equation is **$y = 5x$**.

Using the same method for lines C-H:
line C: **$y = x - 1$** line D: **$y = -3$**
line E: **$y = -\frac{1}{2}x + 2$** line F: **$y = \frac{2}{5}x + 1$**
line G: **$y = \frac{2}{5}x - 4$** line H: **$y = \frac{3}{2}x - \frac{5}{2}$**

4 Using the same method as Q3:
Line A has equation **$y = x + 0.5$**.

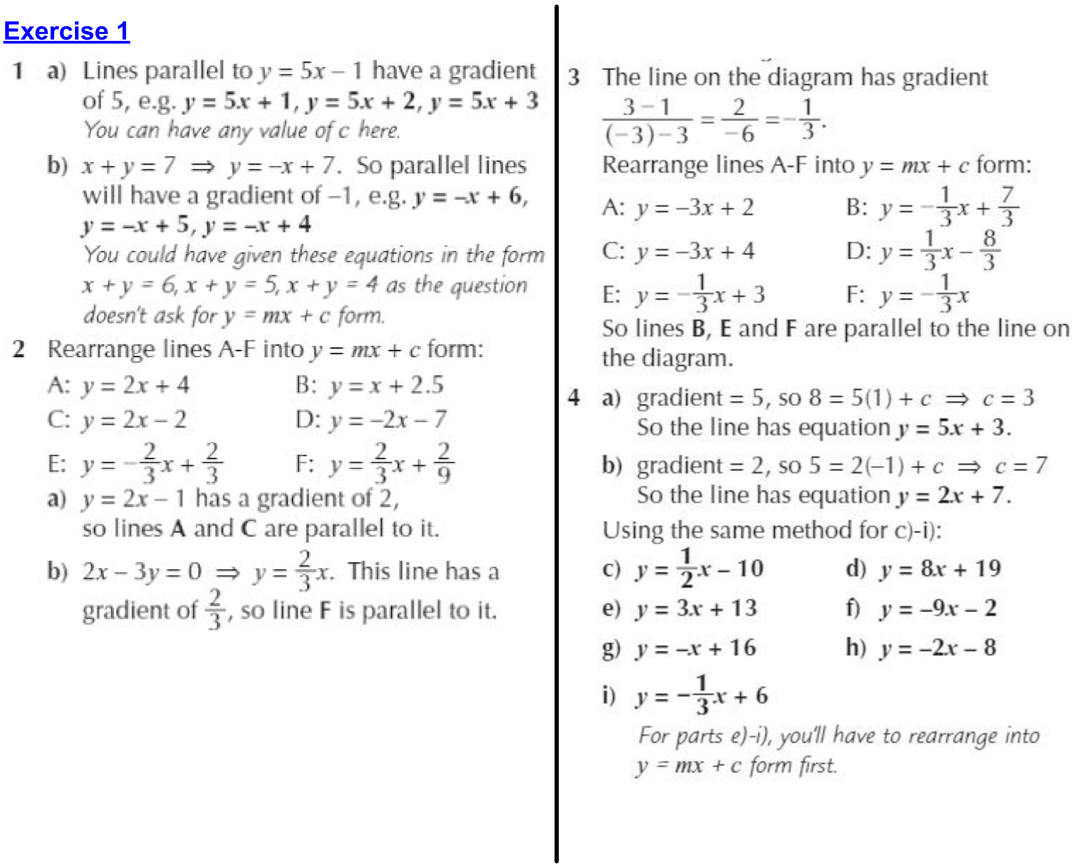
Line B has equation **$y = \frac{1}{10}x + 2$**.

Line C has equation **$y = -\frac{3}{2}x - \frac{1}{2}$**.

Model solutions on

5. Parallel & Perpendicular Lines

Exercise 1



1 a) Lines parallel to $y = 5x - 1$ have a gradient of 5, e.g. $\mathbf{y = 5x + 1}$, $\mathbf{y = 5x + 2}$, $\mathbf{y = 5x + 3}$
You can have any value of c here.

b) $x + y = 7 \Rightarrow y = -x + 7$. So parallel lines will have a gradient of -1, e.g. $\mathbf{y = -x + 6}$, $\mathbf{y = -x + 5}$, $\mathbf{y = -x + 4}$
You could have given these equations in the form $x + y = 6$, $x + y = 5$, $x + y = 4$ as the question doesn't ask for $y = mx + c$ form.

2 Rearrange lines A-F into $y = mx + c$ form:

A: $y = 2x + 4$
B: $y = x + 2.5$
C: $y = 2x - 2$
D: $y = -2x - 7$
E: $y = -\frac{2}{3}x + \frac{2}{3}$
F: $y = \frac{2}{3}x + \frac{2}{9}$

a) $y = 2x - 1$ has a gradient of 2, so lines **A** and **C** are parallel to it.

b) $2x - 3y = 0 \Rightarrow y = \frac{2}{3}x$. This line has a gradient of $\frac{2}{3}$, so line **F** is parallel to it.

3 The line on the diagram has gradient $\frac{3-1}{(-3)-3} = \frac{2}{-6} = -\frac{1}{3}$.
Rearrange lines A-F into $y = mx + c$ form:

A: $y = -3x + 2$
B: $y = -\frac{1}{3}x + \frac{7}{3}$
C: $y = -3x + 4$
D: $y = \frac{1}{3}x - \frac{8}{3}$
E: $y = -\frac{1}{3}x + 3$
F: $y = -\frac{1}{3}x$

So lines **B**, **E** and **F** are parallel to the line on the diagram.

4 a) gradient = 5, so $8 = 5(1) + c \Rightarrow c = 3$
So the line has equation $\mathbf{y = 5x + 3}$.

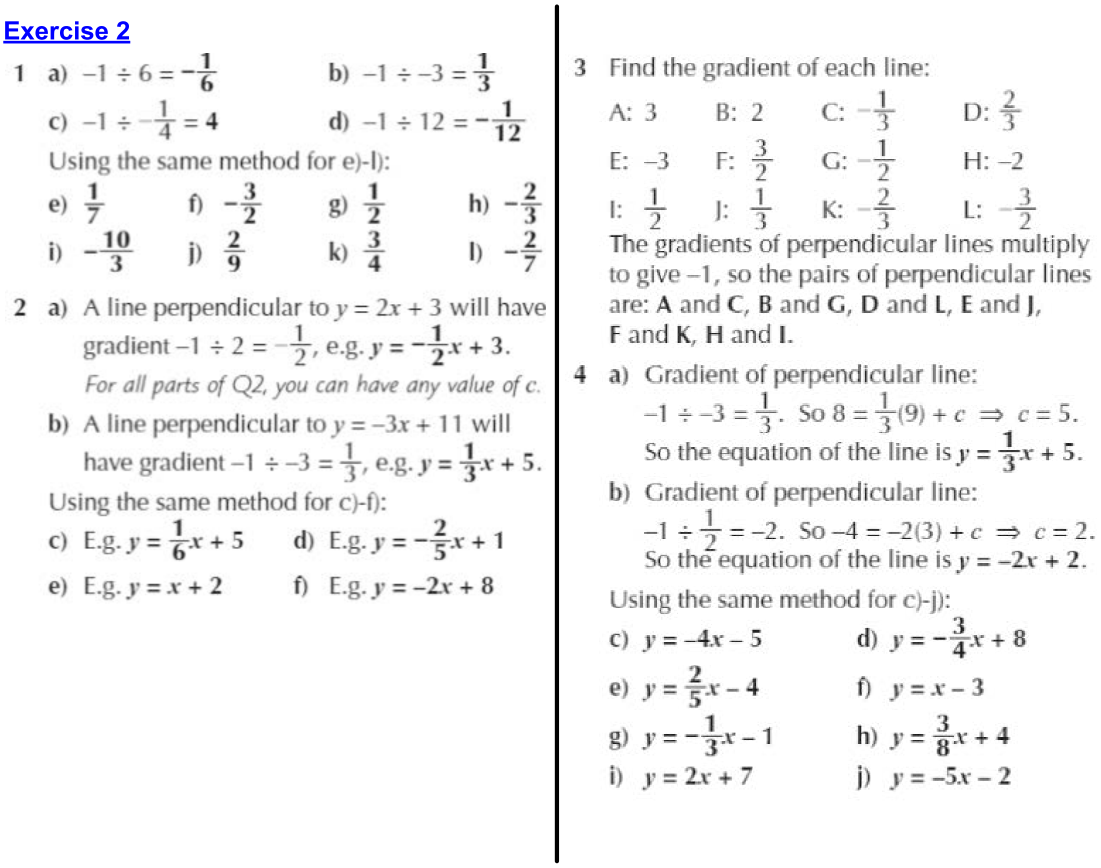
b) gradient = 2, so $5 = 2(-1) + c \Rightarrow c = 7$
So the line has equation $\mathbf{y = 2x + 7}$.

Using the same method for c)-i):

c) $\mathbf{y = \frac{1}{2}x - 10}$
d) $\mathbf{y = 8x + 19}$
e) $\mathbf{y = 3x + 13}$
f) $\mathbf{y = -9x - 2}$
g) $\mathbf{y = -x + 16}$
h) $\mathbf{y = -2x - 8}$
i) $\mathbf{y = -\frac{1}{3}x + 6}$

For parts e)-i), you'll have to rearrange into $y = mx + c$ form first.

Exercise 2



1 a) $-1 \div 6 = \mathbf{-\frac{1}{6}}$
b) $-1 \div -3 = \mathbf{\frac{1}{3}}$
c) $-1 \div -\frac{1}{4} = \mathbf{4}$
d) $-1 \div 12 = \mathbf{-\frac{1}{12}}$

Using the same method for e)-l):

e) $\mathbf{\frac{1}{7}}$
f) $\mathbf{-\frac{3}{2}}$
g) $\mathbf{\frac{1}{2}}$
h) $\mathbf{-\frac{2}{3}}$
i) $\mathbf{-\frac{10}{3}}$
j) $\mathbf{\frac{2}{9}}$
k) $\mathbf{\frac{3}{4}}$
l) $\mathbf{-\frac{2}{7}}$

2 a) A line perpendicular to $y = 2x + 3$ will have gradient $-1 \div 2 = -\frac{1}{2}$, e.g. $\mathbf{y = -\frac{1}{2}x + 3}$.
For all parts of Q2, you can have any value of c.

b) A line perpendicular to $y = -3x + 11$ will have gradient $-1 \div -3 = \frac{1}{3}$, e.g. $\mathbf{y = \frac{1}{3}x + 5}$.

Using the same method for c)-f):

c) E.g. $\mathbf{y = \frac{1}{6}x + 5}$
d) E.g. $\mathbf{y = -\frac{2}{5}x + 1}$
e) E.g. $\mathbf{y = x + 2}$
f) E.g. $\mathbf{y = -2x + 8}$

3 Find the gradient of each line:

A: 3
B: 2
C: $-\frac{1}{3}$
D: $\frac{2}{3}$
E: -3
F: $\frac{3}{2}$
G: $-\frac{1}{2}$
H: -2
I: $\frac{1}{2}$
J: $\frac{1}{3}$
K: $-\frac{2}{3}$
L: $-\frac{3}{2}$

The gradients of perpendicular lines multiply to give -1, so the pairs of perpendicular lines are: **A** and **C**, **B** and **G**, **D** and **L**, **E** and **J**, **F** and **K**, **H** and **I**.

4 a) Gradient of perpendicular line:
$-1 \div -3 = \frac{1}{3}$. So $8 = \frac{1}{3}(9) + c \Rightarrow c = 5$.
So the equation of the line is $\mathbf{y = \frac{1}{3}x + 5}$.

b) Gradient of perpendicular line:
$-1 \div \frac{1}{2} = -2$. So $-4 = -2(3) + c \Rightarrow c = 2$.
So the equation of the line is $\mathbf{y = -2x + 2}$.

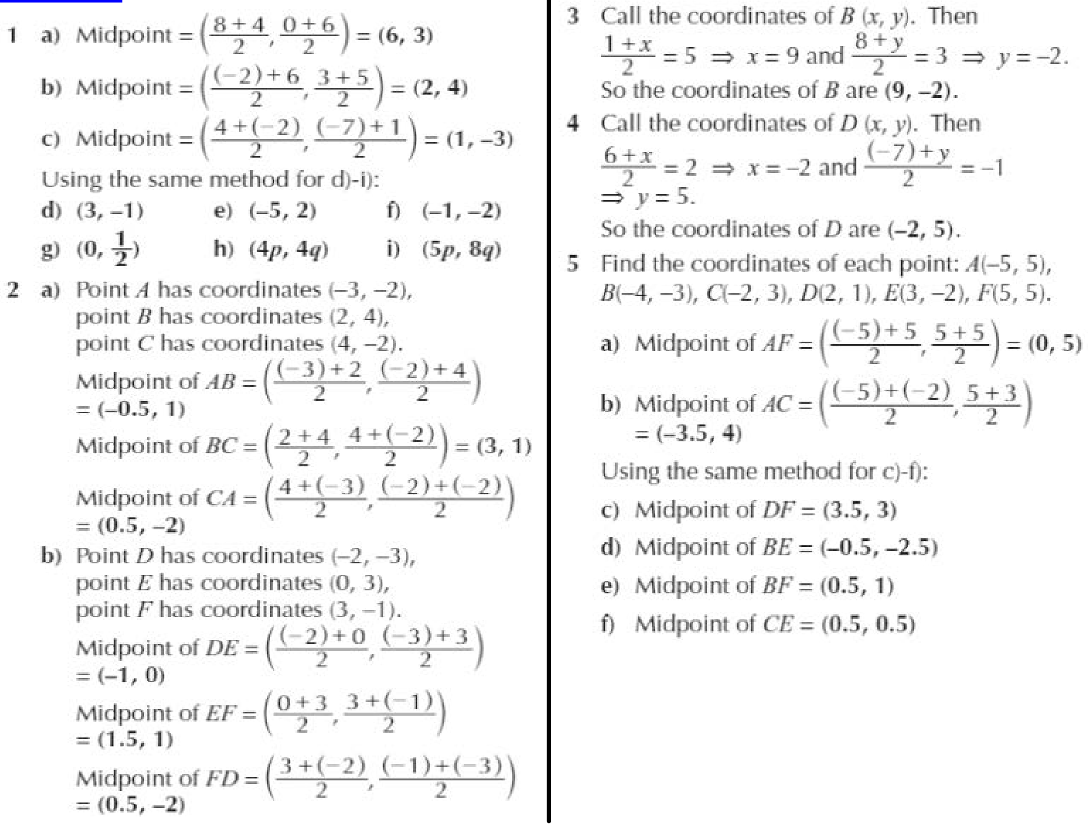
Using the same method for c)-j):

c) $\mathbf{y = -4x - 5}$
d) $\mathbf{y = -\frac{3}{4}x + 8}$
e) $\mathbf{y = \frac{2}{5}x - 4}$
f) $\mathbf{y = x - 3}$
g) $\mathbf{y = -\frac{1}{3}x - 1}$
h) $\mathbf{y = \frac{3}{8}x + 4}$
i) $\mathbf{y = 2x + 7}$
j) $\mathbf{y = -5x - 2}$

Model solutions on

6. Line Segments

Exercise 1



1 a) Midpoint $= \left(\frac{8+4}{2}, \frac{0+6}{2}\right) =$ **(6, 3)**

b) Midpoint $= \left(\frac{(-2)+6}{2}, \frac{3+5}{2}\right) =$ **(2, 4)**

c) Midpoint $= \left(\frac{4+(-2)}{2}, \frac{(-7)+1}{2}\right) =$ **(1, –3)**

Using the same method for d)-i):

d) **(3, –1)** e) **(–5, 2)** f) **(–1, –2)**

g) **(0, $\frac{1}{2}$)** h) **(4*p*, 4*q*)** i) **(5*p*, 8*q*)**

2 a) Point *A* has coordinates (–3, –2),
point *B* has coordinates (2, 4),
point *C* has coordinates (4, –2).

Midpoint of $AB = \left(\frac{(-3)+2}{2}, \frac{(-2)+4}{2}\right)$
= **(–0.5, 1)**

Midpoint of $BC = \left(\frac{2+4}{2}, \frac{4+(-2)}{2}\right) =$ **(3, 1)**

Midpoint of $CA = \left(\frac{4+(-3)}{2}, \frac{(-2)+(-2)}{2}\right)$
= **(0.5, –2)**

b) Point *D* has coordinates (–2, –3),
point *E* has coordinates (0, 3),
point *F* has coordinates (3, –1).

Midpoint of $DE = \left(\frac{(-2)+0}{2}, \frac{(-3)+3}{2}\right)$
= **(–1, 0)**

Midpoint of $EF = \left(\frac{0+3}{2}, \frac{3+(-1)}{2}\right)$
= **(1.5, 1)**

Midpoint of $FD = \left(\frac{3+(-2)}{2}, \frac{(-1)+(-3)}{2}\right)$
= **(0.5, –2)**

3 Call the coordinates of *B* (*x*, *y*). Then
$\frac{1+x}{2} = 5 \Rightarrow x = 9$ and $\frac{8+y}{2} = 3 \Rightarrow y = -2$.
So the coordinates of *B* are **(9, –2)**.

4 Call the coordinates of *D* (*x*, *y*). Then
$\frac{6+x}{2} = 2 \Rightarrow x = -2$ and $\frac{(-7)+y}{2} = -1$
$\Rightarrow y = 5$.
So the coordinates of *D* are **(–2, 5)**.

5 Find the coordinates of each point: *A*(–5, 5),
B(–4, –3), *C*(–2, 3), *D*(2, 1), *E*(3, –2), *F*(5, 5).

a) Midpoint of $AF = \left(\frac{(-5)+5}{2}, \frac{5+5}{2}\right) =$ **(0, 5)**

b) Midpoint of $AC = \left(\frac{(-5)+(-2)}{2}, \frac{5+3}{2}\right)$
= **(–3.5, 4)**

Using the same method for c)-f):

c) Midpoint of *DF* = **(3.5, 3)**

d) Midpoint of *BE* = **(–0.5, –2.5)**

e) Midpoint of *BF* = **(0.5, 1)**

f) Midpoint of *CE* = **(0.5, 0.5)**

Exercise 2

1 **a)** Change in x-coordinates = 5 – 1 = 4
Change in y-coordinates = 9 – 6 = 3
So length = $\sqrt{4^2 + 3^2} = \sqrt{25}$ = **5**

b) Change in x-coordinates = 15 – 11 = 4
Change in y-coordinates = 3 – 8 = –5
So length = $\sqrt{4^2 + (-5)^2} = \sqrt{41}$
= **6.40** (3 s.f.)

Using the same method for c)-l):

c) **3.16** (3 s.f.)
d) **7**
e) **15.6** (3 s.f.)
f) **18.9** (3 s.f.)
g) **8.06** (3 s.f.)
h) **10.0** (3 s.f.)
i) **10.8** (3 s.f.)
j) **8.54** (3 s.f.)
k) **9.85** (3 s.f.)
l) **15.8** (3 s.f.)

2 **a)** Find the coordinates of each point:
$A(-3, -3)$, $B(-2, 3)$, $C(3, 2)$, $D(1, -1)$
Length of $AB = \sqrt{((-3)-(-2))^2 + ((-3)-3)^2}$
$= \sqrt{(-1)^2 + (-6)^2} = \sqrt{37}$ = **6.08** (3 s.f.)
Length of $BC = \sqrt{((-2)-3)^2 + (3-2)^2}$
$= \sqrt{(-5)^2 + 1^2} = \sqrt{26}$ = **5.10** (3 s.f.)
Using the same method for CD and DA:
Length of CD = **3.61** (3 s.f.)
Length of DA = **4.47** (3 s.f.)

b) Find the coordinates of each point:
$E(-4, -1)$, $F(-1, 3)$, $G(3, -2)$, $H(2, -4)$
Length of $EF = \sqrt{((-4)-(-1))^2 + ((-1)-3)^2}$
$= \sqrt{(-3)^2 + (-4)^2} = \sqrt{25}$ = **5**
Length of $FG = \sqrt{((-1)-3)^2 + (3-(-2))^2}$
$= \sqrt{(-4)^2 + 5^2} = \sqrt{41}$ = **6.40** (3 s.f.)
Using the same method for GH and HE:
Length of GH = **2.24** (3 s.f.)
Length of HE = **6.71** (3 s.f.)

Exercise 3

1 **a)** x-difference: 6 – 3 = 3,
y-difference: 6 – (–3) = 9
C lies $\frac{1}{1+2} = \frac{1}{3}$ of the way along AB, so
x: $\frac{1}{3} \times 3 = 1$ and y: $\frac{1}{3} \times 9 = 3$
x-coordinate of C: 3 + 1 = 4,
y-coordinate of C: –3 + 3 = 0,
so C has coordinates **(4, 0)**.

b) x-difference: 9 – (–3) = 12,
y-difference: 1 – 5 = –4
C lies $\frac{3}{3+1} = \frac{3}{4}$ of the way along AB, so
x: $\frac{3}{4} \times 12 = 9$ and y: $\frac{3}{4} \times (-4) = -3$
x-coordinate of C: (–3) + 9 = 6,
y-coordinate of C: 5 + (–3) = 2,
so C has coordinates **(6, 2)**.

Using the same method for c)-d):

c) **(4, 2)**
d) **(–8, –5)**

2 **a)** x-difference between A and B: 2 – 0 = 2,
x-difference between B and C: 6 – 2 = 4
So ratio = 2 : 4 = **1 : 2**
You could have used the y-differences here instead — the ratio would be the same.

b) x-difference between D and E: (–3) – 1 = –4,
x-difference between E and F:
(–4) – (–3) = –1. So ratio = –4 : –1 = **4 : 1**

c) x-difference between G and H: 5 – (–1) = 6,
x-difference between H and I: 14 – 5 = 9
So ratio = 6 : 9 = **2 : 3**

3 **a)** x-difference between S and T: 12 – 6 = 6,
y-difference between S and T: (–4) – 2 = –6
These distances are $\frac{3}{3+2} = \frac{3}{5}$ of the distances from S to U, so x-difference between S and U is (6 ÷ 3) × 5 = 10 and y-difference between S and U is (–6 ÷ 3) × 5 = –10.
So the x-coordinate of U is 6 + 10 = 16 and the y-coordinate of U is 2 + (–10) = –8.
So U has coordinates **(16, –8)**.

b) x-difference between S and T:
18 – (–2) = 20,
y-difference between S and T:
11 – (–4) = 15
These distances are $\frac{5}{5+4} = \frac{5}{9}$ of the distances from S to U, so x-difference between S and U is (20 ÷ 5) × 9 = 36 and y-difference between S and U is (15 ÷ 5) × 9 = 27.
So the x-coordinate of U is (–2) + 36 = 34 and the y-coordinate of U is (–4) + 27 = 23.
So U has coordinates **(34, 23)**.

Model solutions on

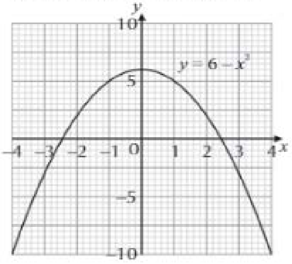
7. Quadratic Graphs

Exercise 1

1 a)

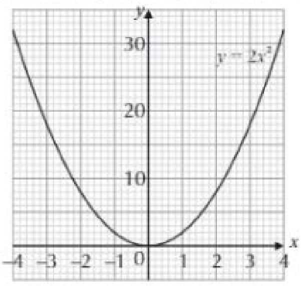
x	−4	−3	−2	−1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$6 - x^2$	−10	−3	2	5	6	5	2	−3	−10

Plot each pair of values (−4, −10), (−3, −3) etc. on a coordinate grid, making sure the axes span the required values, i.e. $-4 \leq x \leq 4$ and $-10 \leq y \leq 6$:



b) Using the same method as part a):

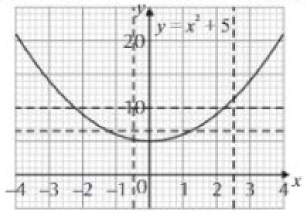
x	−4	−3	−2	−1	0	1	2	3	4
$2x^2$	32	18	8	2	0	2	8	18	32



2 Calculate coordinates in a table of values, e.g:

x	−4	−3	−2	−1	0	1	2	3	4
$x^2 + 5$	21	14	9	6	5	6	9	14	21

Plot them on a set of suitable axes:



a) Draw vertical lines at the given x-values (as shown on graph above) and read off the y-values where these lines meet the graph.

(i) When $x = 2.5$, $\mathbf{y = 11.3}$ (1 d.p.)
(Accept 11.2 to 11.4)

(ii) When $x = -0.5$, $\mathbf{y = 5.3}$ (1 d.p.)
(Accept 5.2 to 5.4)

b) Draw horizontal lines at the given y-values (as shown on graph above) and read off both x-values where these lines meet the graph.

(i) When $y = 6.5$, $\mathbf{x = 1.2}$ and $\mathbf{x = -1.2}$
(both to 1 d.p.)
(Accept 1.1 to 1.3 and −1.3 to −1.1)

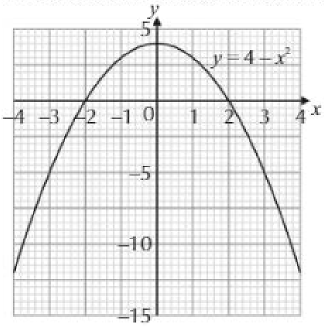
(ii) When $y = 10$, $\mathbf{x = 2.2}$ and $\mathbf{x = -2.2}$
(both to 1 d.p.)
(Accept 2.1 to 2.3 and −2.3 to −2.1)

Exercise 1

3 Calculate coordinates in a table of values, e.g:

x	–4	–3	–2	–1	0	1	2	3	4
$4 – x^2$	–12	–5	0	3	4	3	0	–5	–12

Plot them on a set of suitable axes:

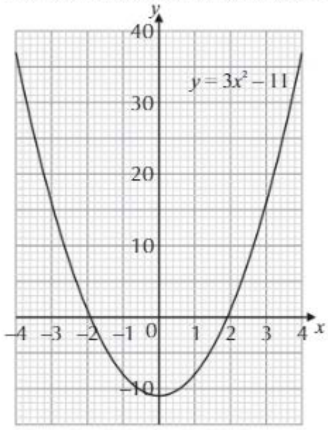


The graph crosses the x-axis at $x = 2$ and $x = –2$.

4 Calculate coordinates in a table of values, e.g:

x	–4	–3	–2	–1	0	1	2	3	4
$3x^2 – 11$	37	16	1	–8	–11	–8	1	16	37

Plot them on a set of suitable axes:



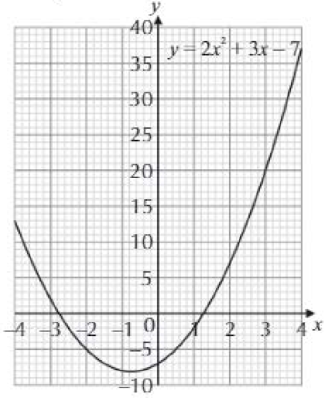
Read off the values where the graph crosses the x-axis: x **= –1.9** and x **= 1.9** (both to 1 d.p.) (Accept –2.0 to –1.8, 1.8 to 2.0)

Exercise 2

1

x	–4	–3	–2	–1	0	1	2	3	4
$2x^2$	**32**	18	**8**	**2**	**0**	**2**	**8**	18	**32**
$+3x$	**–12**	–9	**–6**	**–3**	**0**	**3**	**6**	9	**12**
$–7$	**–7**	–7	**–7**	**–7**	**–7**	**–7**	**–7**	–7	**–7**
$2x^2 + 3x – 7$	**13**	2	**–5**	**–8**	**–7**	**–2**	**7**	20	**37**

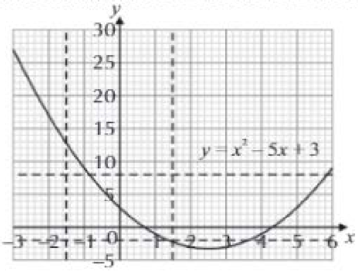
Plot each pair of values (–4, 13), (–3, 2) etc. on a coordinate grid, making sure the axes span the required values, i.e. $–4 \le x \le 4$ and $–8 \le y \le 37$:



2 Calculate coordinates in a table of values, e.g:

x	–3	–2	–1	0	1	2	3	4	5	6
$x^2 – 5x + 3$	27	17	9	3	–1	–3	–3	–1	3	9

Plot them on a set of suitable axes:



a) Draw vertical lines at the given x-values (as shown on graph above) and read off the y-values where these lines meet the graph.

(i) When $x = –1.5$, y **= 12.8** (1 d.p.) (Accept 12.7 to 12.9)

(ii) When $x = 1.5$, y **= –2.3** (1 d.p.) (Accept –2.4 to –2.2)

b) Draw horizontal lines at the given y-values (as shown on graph above) and read off both x-values where these lines meet the graph.

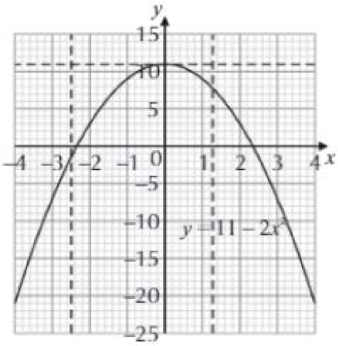
(i) When $y = 8$, x **= –0.9** and x **= 5.9** (both to 1 d.p.) (Accept –1.0 to –0.8, 5.8 to 6.0)

(ii) When $y = –2$, x **= 1.4** and x **= 3.6** (both to 1 d.p.) (Accept 1.3 to 1.5, 3.5 to 3.7)

Exercise 2

3 Using the same method as for question 2:

x	–4	–3	–2	–1	0	1	2	3	4
$11 - 2x^2$	–21	–7	3	9	11	9	3	–7	–21



a) (i) y = **–1.5** (1 d.p.) (Accept –1.6 to –1.4)

(ii) y = **7.9** (1 d.p.) (Accept 7.8 to 8.0)

b) (i) x = **–2.3** and x = **2.3** (both to 1 d.p.) (Accept –2.4 to –2.2, 2.2 to 2.4)

(ii) x = **0**

Exercise 3

1 a) (i) When $x = 0$, $y = (0 - 1)(0 + 1) = -1$. So the y-intercept is **(0, –1)**. When $(x - 1)(x + 1) = 0$, $x = 1$ or $x = -1$. So the x-intercepts are **(–1, 0)** and **(1, 0)**.

(ii) The x-coordinate of the turning point is halfway between the x-intercepts i.e. $(-1 + 1) \div 2 = 0$. When $x = 0$, $y = (0 - 1)(0 + 1) = -1$. So the turning point is at **(0, –1)**.

Here, the turning point is also the y-intercept.

Using the same method for b)-c):

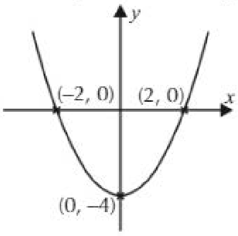
b) (i) The y-intercept is **(0, 7)**. The x-intercepts are **(–7, 0)** and **(–1, 0)**.

(ii) The turning point is at **(–4, –9)**.

c) (i) First factorise: $y = (x + 10)(x + 6)$. The y-intercept is **(0, 60)**. The x-intercepts are **(–10, 0)** and **(–6, 0)**.

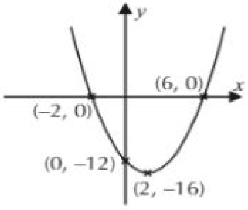
(ii) The turning point is at **(–8, –4)**.

2 a) When $x = 0$, $y = 0^2 - 4 = -4$. So the y-intercept is (0, –4). $y = x^2 - 4$ factorises to $y = (x - 2)(x + 2)$. When $(x - 2)(x + 2) = 0$, $x = 2$ or $x = -2$. So the x-intercepts are (–2, 0) and (2, 0). The x-coordinate of the turning point is halfway between the x-intercepts i.e. $(-2 + 2) \div 2 = 0$. When $x = 0$, $y = 0^2 - 4 = -4$. So the turning point is at (0, –4). The x^2 term is positive, so sketch a u-shaped curve passing through the intercepts and turning point:



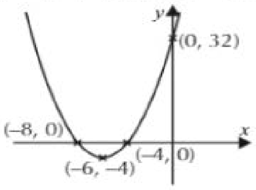
Exercise 3

b) When $x = 0$, $y = 0^2 - (4 \times 0) - 12 = -12$.
So the y-intercept is $(0, -12)$.
$y = x^2 - 4x - 12 = (x - 6)(x + 2)$.
When $(x - 6)(x + 2) = 0$, $x = 6$ or $x = -2$.
So the x-intercepts are $(-2, 0)$ and $(6, 0)$.
The x-coordinate of the turning point is halfway between the x-intercepts i.e. $(-2 + 6) \div 2 = 2$. When $x = 2$, $y = 2^2 - (4 \times 2) - 12 = -16$. So the turning point is at $(2, -16)$. The x^2 term is positive, so sketch a u-shaped curve passing through the intercepts and turning point:

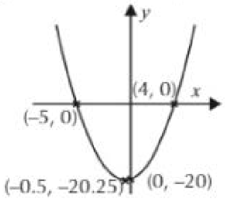


Using the same method for c)-i):

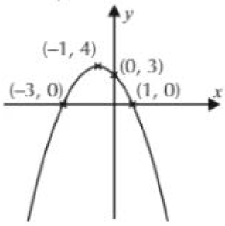
c) $y = x^2 + 12x + 32 = (x + 4)(x + 8)$



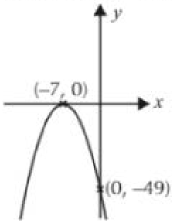
d) $y = x^2 + x - 20 = (x + 5)(x - 4)$



e) $y = -x^2 - 2x + 3 = (x + 3)(1 - x)$
Here, the x^2 term is negative, so it's an n-shaped curve:

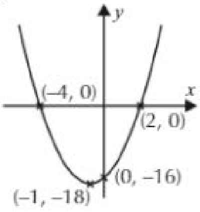


f) $y = -x^2 - 14x - 49 = -(x + 7)^2$
or $(x + 7)(-x - 7)$

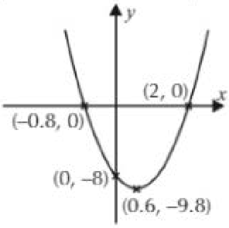


Exercise 3

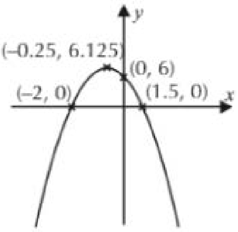
g) $y = 2x^2 + 4x - 16 = 2(x + 4)(x - 2)$



h) $y = 5x^2 - 6x - 8 = (5x + 4)(x - 2)$



i) $y = -2x^2 - x + 6 = (3 - 2x)(x + 2)$



Exercise 4

1 a) (i) When $x = 0$, $y = (0 + 5)^2 - 9 = 16$. So the y-intercept is **(0, 16)**.

(ii) The turning point is when $(x + 5) = 0$ $\Rightarrow x = -5$ and $y = 0^2 - 9 = -9$. So the turning point is at **(–5, –9)**.

You can just read this off from the completed square form of the equation.

Using the same method for b)-c):

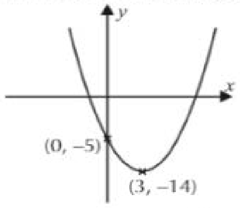
b) (i) The y-intercept is **(0, –21)**.

(ii) The turning point is at **(3, –30)**.

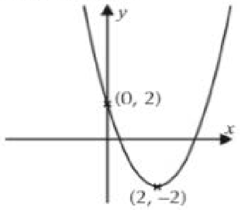
c) (i) The y-intercept is **(0, 3)**.

(ii) The turning point is at **(4, –13)**.

2 a) When $x = 0$, $y = 0^2 - (6 \times 0) - 5 = -5$. So the y-intercept is (0, –5). Completing the square: $y = x^2 - 6x - 5 = (x - 3)^2 - 14$, so the turning point is at (3, –14). The x^2 term is positive, so sketch a u-shaped curve passing through the coordinates for the intercept and turning point:



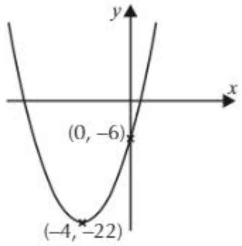
b) When $x = 0$, $y = 0^2 - (4 \times 0) + 2 = 2$. So the y-intercept is (0, 2). Completing the square: $y = x^2 - 4x + 2 = (x - 2)^2 - 2$, so the turning point is at (2, –2). The x^2 term is positive, so sketch a u-shaped curve passing through the coordinates for the intercept and turning point:



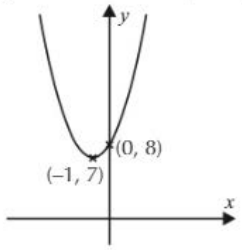
Exercise 4

Using the same method for c)-f):

c) $y = x^2 + 8x - 6 = (x + 4)^2 - 22$

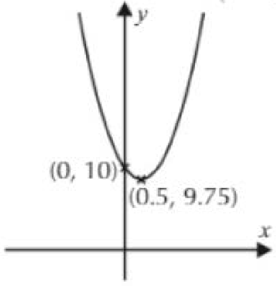


d) $y = x^2 + 2x + 8 = (x + 1)^2 + 7$



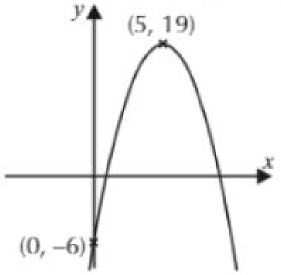
This graph does not cross the x-axis because the turning point is above the x-axis on a u-shaped quadratic.

e) $y = x^2 - x + 10 = \left(x - \frac{1}{2}\right)^2 + 9\frac{3}{4}$



f) $y = -x^2 + 10x - 6 = 19 - (x - 5)^2$

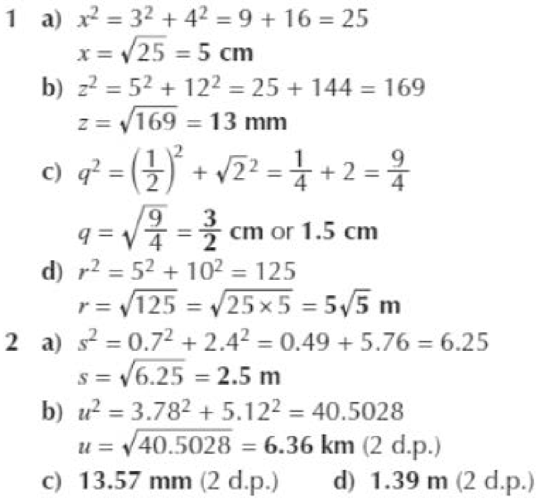
The x^2 term is negative, so it's n-shaped:



Model solutions on

1. Pythagoras' Theorem

Exercise 1



1 **a)** $x^2 = 3^2 + 4^2 = 9 + 16 = 25$

$x = \sqrt{25} = \mathbf{5\ cm}$

b) $z^2 = 5^2 + 12^2 = 25 + 144 = 169$

$z = \sqrt{169} = \mathbf{13\ mm}$

c) $q^2 = \left(\frac{1}{2}\right)^2 + \sqrt{2}^2 = \frac{1}{4} + 2 = \frac{9}{4}$

$q = \sqrt{\frac{9}{4}} = \mathbf{\frac{3}{2}}$ cm or **1.5 cm**

d) $r^2 = 5^2 + 10^2 = 125$

$r = \sqrt{125} = \sqrt{25 \times 5} = \mathbf{5\sqrt{5}\ m}$

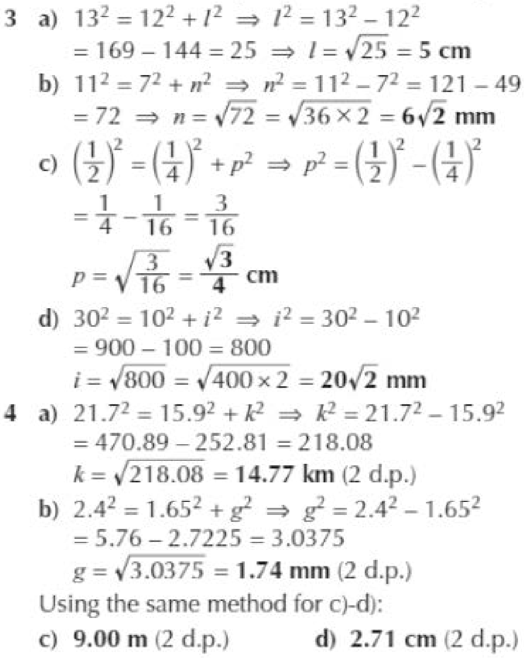
2 **a)** $s^2 = 0.7^2 + 2.4^2 = 0.49 + 5.76 = 6.25$

$s = \sqrt{6.25} = \mathbf{2.5\ m}$

b) $u^2 = 3.78^2 + 5.12^2 = 40.5028$

$u = \sqrt{40.5028} = \mathbf{6.36\ km}$ (2 d.p.)

c) **13.57 mm** (2 d.p.) **d)** **1.39 m** (2 d.p.)



3 **a)** $13^2 = 12^2 + l^2 \Rightarrow l^2 = 13^2 - 12^2$

$= 169 - 144 = 25 \Rightarrow l = \sqrt{25} = \mathbf{5\ cm}$

b) $11^2 = 7^2 + n^2 \Rightarrow n^2 = 11^2 - 7^2 = 121 - 49$

$= 72 \Rightarrow n = \sqrt{72} = \sqrt{36 \times 2} = \mathbf{6\sqrt{2}\ mm}$

c) $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2 + p^2 \Rightarrow p^2 = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2$

$= \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

$p = \sqrt{\frac{3}{16}} = \mathbf{\frac{\sqrt{3}}{4}}$ cm

d) $30^2 = 10^2 + i^2 \Rightarrow i^2 = 30^2 - 10^2$

$= 900 - 100 = 800$

$i = \sqrt{800} = \sqrt{400 \times 2} = \mathbf{20\sqrt{2}\ mm}$

4 **a)** $21.7^2 = 15.9^2 + k^2 \Rightarrow k^2 = 21.7^2 - 15.9^2$

$= 470.89 - 252.81 = 218.08$

$k = \sqrt{218.08} = \mathbf{14.77\ km}$ (2 d.p.)

b) $2.4^2 = 1.65^2 + g^2 \Rightarrow g^2 = 2.4^2 - 1.65^2$

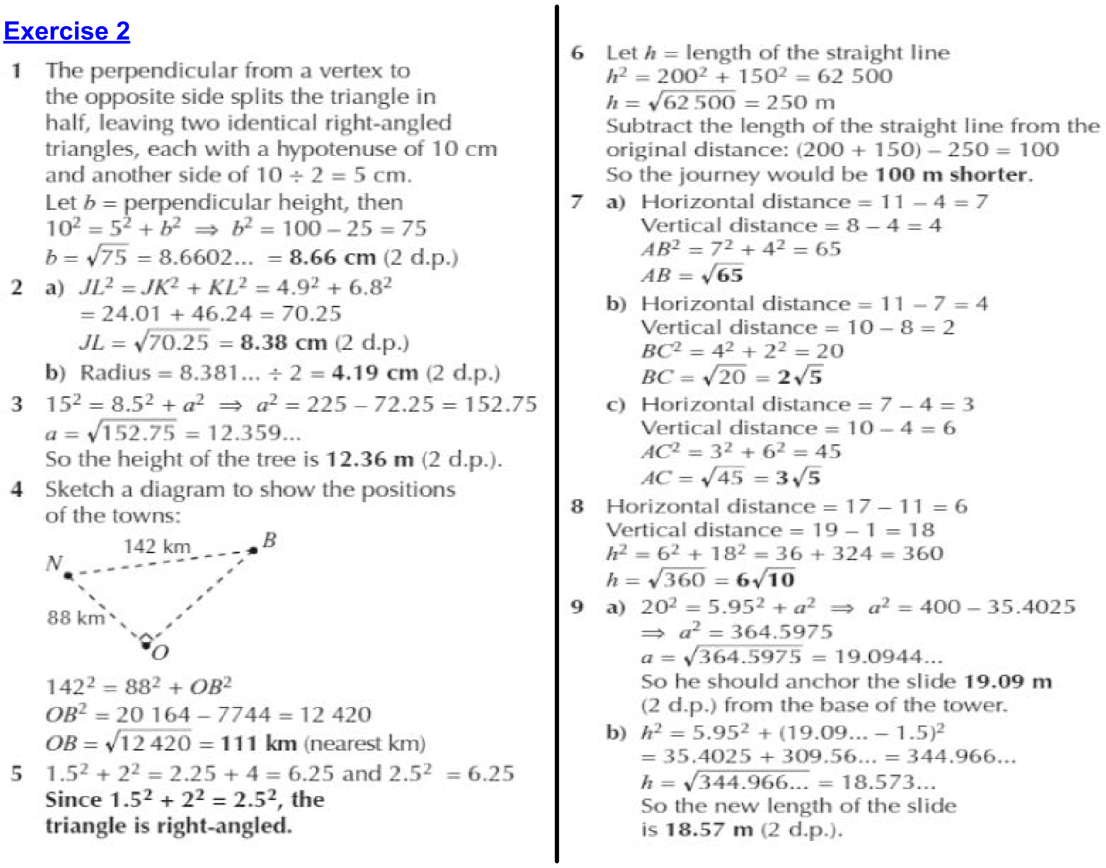
$= 5.76 - 2.7225 = 3.0375$

$g = \sqrt{3.0375} = \mathbf{1.74\ mm}$ (2 d.p.)

Using the same method for c)-d):

c) **9.00 m** (2 d.p.) **d)** **2.71 cm** (2 d.p.)

Exercise 2



1 The perpendicular from a vertex to the opposite side splits the triangle in half, leaving two identical right-angled triangles, each with a hypotenuse of 10 cm and another side of 10 ÷ 2 = 5 cm.
Let h = perpendicular height, then
$10^2 = 5^2 + h^2 \Rightarrow h^2 = 100 - 25 = 75$
$h = \sqrt{75} = 8.6602... = \mathbf{8.66}$ **cm** (2 d.p.)

2 **a)** $JL^2 = JK^2 + KL^2 = 4.9^2 + 6.8^2$
$= 24.01 + 46.24 = 70.25$
$JL = \sqrt{70.25} = \mathbf{8.38}$ **cm** (2 d.p.)

b) Radius = 8.381... ÷ 2 = **4.19 cm** (2 d.p.)

3 $15^2 = 8.5^2 + a^2 \Rightarrow a^2 = 225 - 72.25 = 152.75$
$a = \sqrt{152.75} = 12.359...$
So the height of the tree is **12.36 m** (2 d.p.).

4 Sketch a diagram to show the positions of the towns:

$142^2 = 88^2 + OB^2$
$OB^2 = 20\,164 - 7744 = 12\,420$
$OB = \sqrt{12\,420} = \mathbf{111}$ **km** (nearest km)

5 $1.5^2 + 2^2 = 2.25 + 4 = 6.25$ and $2.5^2 = 6.25$
Since $1.5^2 + 2^2 = 2.5^2$, the triangle is right-angled.

6 Let h = length of the straight line
$h^2 = 200^2 + 150^2 = 62\,500$
$h = \sqrt{62\,500} = 250$ m
Subtract the length of the straight line from the original distance: (200 + 150) – 250 = 100
So the journey would be **100 m shorter**.

7 **a)** Horizontal distance = 11 – 4 = 7
Vertical distance = 8 – 4 = 4
$AB^2 = 7^2 + 4^2 = 65$
$AB = \mathbf{\sqrt{65}}$

b) Horizontal distance = 11 – 7 = 4
Vertical distance = 10 – 8 = 2
$BC^2 = 4^2 + 2^2 = 20$
$BC = \sqrt{20} = \mathbf{2\sqrt{5}}$

c) Horizontal distance = 7 – 4 = 3
Vertical distance = 10 – 4 = 6
$AC^2 = 3^2 + 6^2 = 45$
$AC = \sqrt{45} = \mathbf{3\sqrt{5}}$

8 Horizontal distance = 17 – 11 = 6
Vertical distance = 19 – 1 = 18
$h^2 = 6^2 + 18^2 = 36 + 324 = 360$
$h = \sqrt{360} = \mathbf{6\sqrt{10}}$

9 **a)** $20^2 = 5.95^2 + a^2 \Rightarrow a^2 = 400 - 35.4025$
$\Rightarrow a^2 = 364.5975$
$a = \sqrt{364.5975} = 19.0944...$
So he should anchor the slide **19.09 m** (2 d.p.) from the base of the tower.

b) $h^2 = 5.95^2 + (19.09... - 1.5)^2$
$= 35.4025 + 309.56... = 344.966...$
$h = \sqrt{344.966...} = 18.573...$
So the new length of the slide is **18.57 m** (2 d.p.).

Model solutions on

2. Pythagoras' Theorem in 3D

Exercise 1

1 a) $BD^2 = AB^2 + AD^2 = 4^2 + 8^2 = 80$

$BD = \sqrt{80} =$ **$4\sqrt{5}$ m**

b) $FD^2 = BD^2 + BF^2 = 80 + 3^2 = 89$

$FD =$ **$\sqrt{89}$ m**

2 a) $PR^2 = PS^2 + RS^2 = 12^2 + 5^2 = 169$

$PR = \sqrt{169} =$ **13 mm**

b) $RT^2 = PR^2 + PT^2 = 169 + 9^2 = 250$

$RT = \sqrt{250} =$ **$5\sqrt{10}$ mm**

3 Diameter = 4.5 × 2 = 9 cm

$XY^2 = 9^2 + 25^2 = 81 + 625 = 706$

$XY = \sqrt{706} =$ **26.6 cm** (3 s.f.)

4 $d^2 = 2.5^2 + 3.8^2 + 9.4^2 = 6.25 + 14.44 + 88.36$

$= 109.05$

$d = \sqrt{109.05} =$ **10.44 m** (2 d.p.)

5 a) $QS^2 = PQ^2 + PS^2 = 14^2 + 25^2 = 821$

$QS = \sqrt{821} =$ **28.7 mm** (3 s.f.)

b) $ST^2 = QS^2 + QT^2 = 821 + 9^2 = 902$

$ST = \sqrt{902} =$ **30.0 mm** (3 s.f.)

6 $d^2 = 5^2 + 5^2 + 5^2 = 75$

$d = \sqrt{75} =$ **8.66 m** (3 s.f.)

7 $d^2 = 16.5^2 + 4.8^2 + 2^2 = 299.29$

$d = \sqrt{299.29} = 17.3$

So the longest pencil that can fit would be **17.3 cm** long.

8 Diameter = 6 × 2 = 12 cm

$h^2 = 12^2 + 28^2 = 144 + 784 = 928$

$h = \sqrt{928} = 30.463...$

So the longest stick of dried spaghetti that can fit would be **30.5 cm** (3 s.f.).

9 Let d = diagonal of base

$d^2 = 4.8^2 + 4.8^2 = 46.08$

$d = \sqrt{46.08} = 6.788...$ cm

$6.788.. \div 2 = 3.39...$

Let V = vertical height of pyramid

$11.2^2 = 3.39...^2 + V^2$

$V^2 = 125.44 - 11.52 = 113.92$

$V = \sqrt{113.92} =$ **10.7 cm** (3 s.f.)

Exercise 1

10 Let d = diagonal of base

$d^2 = 3.2^2 + 3.2^2 = 20.48$

$d = \sqrt{20.48} = 4.525...$ m

$4.525... \div 2 = 2.262...$

Let s = length of sloped edge

$s^2 = 2.262...^2 + 9.2^2 = 89.76$

$s = \sqrt{89.76} =$ **9.47 m** (3 s.f.)

11 a) $LN^2 = LP^2 + NP^2$

$13^2 = 3.5^2 + NP^2$

$NP^2 = 13^2 - 3.5^2 = 169 - 12.25 = 156.75$

$NP = \sqrt{156.75} =$ **12.5 m** (3 s.f.)

b) $JL^2 = JM^2 + LM^2 = 7^2 + 7^2 = 98$

$JL = \sqrt{98}$

$OJ = JL \div 2 = \sqrt{98} \div 2 =$ **4.95 m** (3 s.f.)

You could find OJ directly by using the fact that the perpendicular distance from O to each edge of the square is 3.5 m ($OJ^2 = 3.5^2 + 3.5^2$).

c) $NJ^2 = OJ^2 + ON^2$

$ON^2 = NJ^2 - OJ^2 = 13^2 - 4.949...^2$

$= 169 - 24.5 = 144.5$

$ON = \sqrt{144.5} =$ **12.0 m** (3 s.f.)

12 a) $XZ^2 = OX^2 + OZ^2$

$OX^2 = XZ^2 - OZ^2 = 17^2 - 15^2 = 64$

$OX = \sqrt{64} = 8$ m

$VX = OX \times 2 =$ **16 m**

b) $VXWY$ is a square so $VY = XY$, which means area $= VY^2$

Using Pythagoras' theorem:

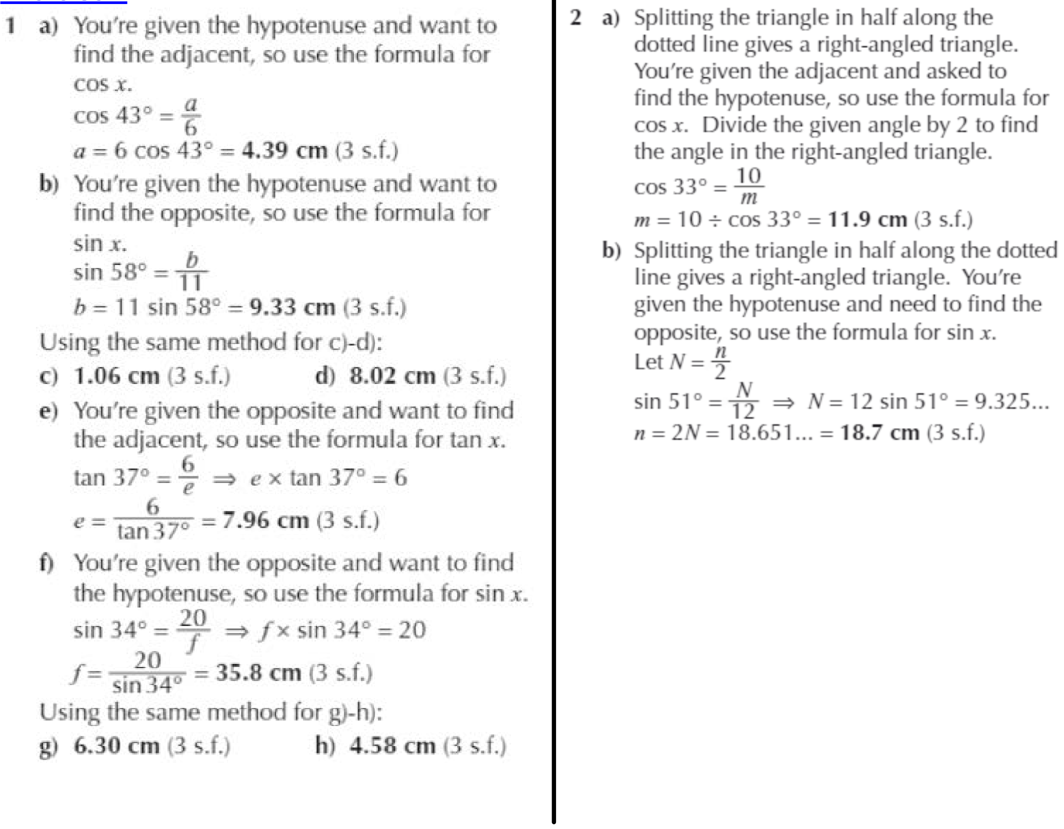
$VX^2 = VY^2 + XY^2 = 2VY^2$

$VY^2 = VX^2 \div 2 = 16^2 \div 2 =$ **128 m²**

Model solutions on

3. Trigonometry - Sin, Cos and Tan

Exercise 1



1 **a)** You're given the hypotenuse and want to find the adjacent, so use the formula for cos x.

$\cos 43° = \frac{a}{6}$

$a = 6 \cos 43° =$ **4.39 cm** (3 s.f.)

b) You're given the hypotenuse and want to find the opposite, so use the formula for sin x.

$\sin 58° = \frac{b}{11}$

$b = 11 \sin 58° =$ **9.33 cm** (3 s.f.)

Using the same method for c)-d):

c) **1.06 cm** (3 s.f.) **d)** **8.02 cm** (3 s.f.)

e) You're given the opposite and want to find the adjacent, so use the formula for tan x.

$\tan 37° = \frac{6}{e} \Rightarrow e \times \tan 37° = 6$

$e = \frac{6}{\tan 37°} =$ **7.96 cm** (3 s.f.)

f) You're given the opposite and want to find the hypotenuse, so use the formula for sin x.

$\sin 34° = \frac{20}{f} \Rightarrow f \times \sin 34° = 20$

$f = \frac{20}{\sin 34°} =$ **35.8 cm** (3 s.f.)

Using the same method for g)-h):

g) **6.30 cm** (3 s.f.) **h)** **4.58 cm** (3 s.f.)

2 **a)** Splitting the triangle in half along the dotted line gives a right-angled triangle. You're given the adjacent and asked to find the hypotenuse, so use the formula for cos x. Divide the given angle by 2 to find the angle in the right-angled triangle.

$\cos 33° = \frac{10}{m}$

$m = 10 \div \cos 33° =$ **11.9 cm** (3 s.f.)

b) Splitting the triangle in half along the dotted line gives a right-angled triangle. You're given the hypotenuse and need to find the opposite, so use the formula for sin x.

Let $N = \frac{n}{2}$

$\sin 51° = \frac{N}{12} \Rightarrow N = 12 \sin 51° = 9.325...$

$n = 2N = 18.651... =$ **18.7 cm** (3 s.f.)

Exercise 2

1 **a)** You're given the adjacent and hypotenuse, so use the formula for cos x.

$\cos a = \frac{3}{8}$

$a = \cos^{-1}\left(\frac{3}{8}\right) = \mathbf{68.0°}$ (1 d.p.)

b) You're given the opposite and hypotenuse, so use the formula for sin x.

$\sin b = \frac{10}{14}$

$b = \sin^{-1}\left(\frac{10}{14}\right) = \mathbf{45.6°}$ (1 d.p.)

Using the same method for c)-f):

c) **47.5°** (1 d.p.) **d)** **80.4°** (1 d.p.)

e) **33.7°** (1 d.p.) **f)** **60.1°** (1 d.p.)

2 **a)** You're given the opposite and hypotenuse, so use the formula for sin x.

$\sin m = \frac{8}{24} \Rightarrow m = \sin^{-1}\left(\frac{8}{24}\right) = 19.471...$

So the angle of elevation of the slide is **19.5°** (1 d.p.).

b) Using alternate angles, q is the same as the angle opposite the ladder. So you know the opposite and the adjacent so use the formula for tan x.

$\tan q = \frac{4}{5.5}$

$q = \tan^{-1}\left(\frac{4}{5.5}\right) = 36.027...$

So the angle of depression from the top of the slide is **36.0°** (1 d.p.).

3 **a)** Splitting the triangle in half along the dotted line gives a right-angled triangle. Let $Z = \frac{z}{2}$

You're given the adjacent and the hypotenuse, so use the formula for cos x.

$\cos Z = \frac{14}{18} \Rightarrow Z = \cos^{-1}\left(\frac{14}{18}\right) = 38.942...$

$z = 2Z = \mathbf{77.9°}$ (1 d.p.)

b) In this part, use the properties of rectangles and triangles to find the lengths of missing sides.

j: You know the opposite and adjacent, so use the formula for tan x.

$\tan j = \frac{6}{2} \Rightarrow j = \tan^{-1}\left(\frac{6}{2}\right) = \mathbf{71.6°}$ (1 d.p.)

k: You know the opposite and adjacent, so use the formula for tan x.

$\tan k = \frac{10}{6}$

$k = \tan^{-1}\left(\frac{10}{6}\right) = \mathbf{59.0°}$ (1 d.p.)

Using the same method for angles l-n:

$l = \mathbf{31.0°}$, $m = \mathbf{45.6°}$, $n = \mathbf{70.5°}$

Exercise 3

1 Using the common values:

a) $\sin a = \frac{\sqrt{3}}{2} \Rightarrow a = \mathbf{60°}$

b) $\cos b = \frac{1}{\sqrt{2}} \Rightarrow b = \mathbf{45°}$

c) $\tan c = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow c = \mathbf{60°}$

d) $\sin d = \frac{4}{8} = \frac{1}{2} \Rightarrow d = \mathbf{30°}$

2 **a)** $\sin 30° = \frac{e}{2}$

$\Rightarrow e = 2 \sin 30° = 2 \times \frac{1}{2} = \mathbf{1\ m}$

b) $\cos 60° = \frac{f}{2}$

$\Rightarrow f = 2 \cos 60° = 2 \times \frac{1}{2} = \mathbf{1\ cm}$

c) $\tan 45° = \frac{8}{g}$

$\Rightarrow g = \frac{8}{\tan 45°} = \frac{8}{1} = \mathbf{8\ mm}$

d) $\tan 30° = \frac{1}{h}$

$\Rightarrow h = \frac{1}{\tan 30°} = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} = \mathbf{\sqrt{3}\ m}$

3 **a)** $\tan 45° = 1$, $\sin 60° = \frac{\sqrt{3}}{2}$

$1 + \frac{\sqrt{3}}{2} = \frac{2}{2} + \frac{\sqrt{3}}{2} = \mathbf{\frac{2+\sqrt{3}}{2}}$

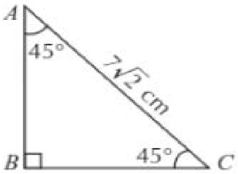
b) $\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \mathbf{\sqrt{2}}$

c) $\tan 30° = \frac{1}{\sqrt{3}}$, $\tan 60° = \sqrt{3}$

$\frac{1}{\sqrt{3}} + \sqrt{3} = \frac{1}{\sqrt{3}} + \frac{3}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \mathbf{\frac{4\sqrt{3}}{3}}$

4



The triangle is right-angled with acute angles of 45°. You're given the hypotenuse and need to find the opposite or adjacent, so use the formula for sin x or cos x. E.g.

$\sin 45° = \frac{AB}{7\sqrt{2}}$

$\Rightarrow AB = 7\sqrt{2} \sin 45° = 7\sqrt{2} \times \frac{1}{\sqrt{2}} = \mathbf{7\ cm}$

You could also solve this using Pythagoras' theorem. $AB = BC$ *so* $AB^2 + AB^2 = 2AB^2 = (7\sqrt{2})^2$.

5 E.g. Angle $EDF = 60°$ since the triangle is equilateral. Then use the formula for sin x:

$\sin 60° = \frac{4}{ED} \Rightarrow \frac{\sqrt{3}}{2} = \frac{4}{ED}$

$\Rightarrow ED\ (= DF = ED) = \frac{4}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{8}{\sqrt{3}} = \mathbf{\frac{8\sqrt{3}}{3}\ mm}$

Model solutions on

4. The Sine and Cosine Rules

Exercise 1

1 **a)** $\frac{a}{\sin 50°} = \frac{16}{\sin 80°}$

$a = \frac{16 \sin 50°}{\sin 80°} = \mathbf{12.4\ cm}$ (3 s.f.)

b) $\frac{b}{\sin 122°} = \frac{3}{\sin 17°}$

$b = \frac{3 \sin 122°}{\sin 17°} = \mathbf{8.70\ in}$ (3 s.f.)

Using the same method for c):

c) **5.59 mm** (3 s.f.)

2 **a)** $\frac{\sin g}{13} = \frac{\sin 27°}{11}$

$g = \sin^{-1}\left(\frac{13 \sin 27°}{11}\right) = \mathbf{32.4°}$ (1 d.p.)

b) $\frac{\sin h}{9} = \frac{\sin 102°}{16}$

$h = \sin^{-1}\left(\frac{9 \sin 102°}{16}\right) = \mathbf{33.4°}$ (1 d.p.)

Using the same method for c)-f):

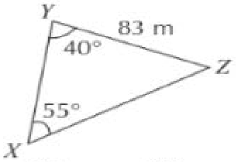
c) **38.8°** (1 d.p.)

d) **67.0°** (1 d.p.)

e) **44.4°** (1 d.p.)

f) **34.1°** (1 d.p.)

3 **a)**



$\frac{YZ}{\sin YXZ} = \frac{XZ}{\sin XYZ} \Rightarrow \frac{83}{\sin 55°} = \frac{XZ}{\sin 40°}$

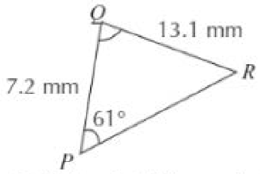
$XZ = \frac{83 \sin 40°}{\sin 55°} = \mathbf{65.1\ m}$ (3 s.f.)

b) $XZY = 180° - (55° + 40°) = 85°$

$\frac{YZ}{\sin YXZ} = \frac{XY}{\sin XZY} \Rightarrow \frac{83}{\sin 55°} = \frac{XY}{\sin 85°}$

$XY = \frac{83 \sin 85°}{\sin 55°} = \mathbf{101\ m}$ (3 s.f.)

4

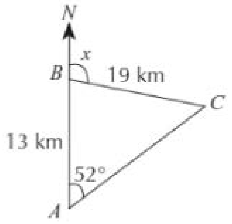


$\frac{\sin RPQ}{QR} = \frac{\sin PRQ}{PQ} \Rightarrow \frac{\sin 61°}{13.1} = \frac{\sin PRQ}{7.2}$

$PRQ = \sin^{-1}\left(\frac{7.2 \sin 61°}{13.1}\right) = 28.731...°$

$PQR = 180° - 61° - 28.731...° = \mathbf{90.3°}$ (3 s.f.)

5



$\frac{\sin 52°}{19} = \frac{\sin ACB}{13} ACB = \sin^{-1}\left(\frac{13 \sin 52°}{19}\right) =$

$32.626...°$

$ABC = 180° - 52° - 32.626...° = 95.373...°$

$x = 180° - 95.373...° = 84.626...°$

$= \mathbf{085°}$ (to the nearest degree)

The answer is a bearing, so you need to give it as three figures.

Exercise 2

1 **a)** $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 7^2 + 3^2 - (2 \times 7 \times 3 \times \cos 66°) = 40.9...$

$a = \sqrt{40.9...} =$ **6.40 cm** (3 s.f.)

b) $b^2 = 6.5^2 + 8^2 - (2 \times 6.5 \times 8 \times \cos 42°)$

$b^2 = 28.9...$

$b = \sqrt{28.9...} =$ **5.38 cm** (3 s.f.)

Using the same method for c)-f):

c) **18.5 m** (3 s.f.) **d)** **4.65 in** (3 s.f.)

e) **3.63 cm** (3 s.f.) **f)** **4.59 m** (3 s.f.)

2 **a)** $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos p = \frac{5^2 + 7^2 - 8^2}{2 \times 5 \times 7}$

$p = \cos^{-1}\left(\frac{5^2 + 7^2 - 8^2}{2 \times 5 \times 7}\right) =$ **81.8°** (1 d.p.)

b) $\cos q = \frac{3^2 + 11^2 - 10^2}{2 \times 3 \times 11}$

$q = \cos^{-1}\left(\frac{3^2 + 11^2 - 10^2}{2 \times 3 \times 11}\right) =$ **63.0°** (1 d.p.)

Using the same method for c)-f):

c) **54.0°** (1 d.p.) **d)** **36.7°** (1 d.p.)

e) **61.1°** (1 d.p.) **f)** **115.7°** (1 d.p.)

3 $\cos XYZ = \frac{XY^2 + YZ^2 - XZ^2}{2 \times XY \times YZ} = \frac{67^2 + 78^2 - 99^2}{2 \times 67 \times 78}$

$XYZ = \cos^{-1}\left(\frac{67^2 + 78^2 - 99^2}{2 \times 67 \times 78}\right) =$ **85.8°** (3 s.f.)

4 Since this question is non-calculator, you'll have to use the common trig values from pages 326 and 327.

a) $x^2 = 5^2 + 8^2 - (2 \times 5 \times 8 \times \cos 60°)$

$x^2 = 25 + 64 - 80 \times \frac{1}{2} = 89 - 40 = 49$

$x = \sqrt{49} =$ **7 m**

b) $y^2 = 3^2 + (3\sqrt{2})^2 - (2 \times 3 \times 3\sqrt{2} \times \cos 45°)$

$y^2 = 9 + 18 - 18\sqrt{2} \times \frac{1}{\sqrt{2}} = 9 + 18 - 18 = 9$

$y = \sqrt{9} =$ **3 m**

c) $z^2 = 2^2 + (\sqrt{12})^2 - (2 \times 2 \times \sqrt{12} \times \cos 30°)$

$z^2 = 4 + 12 - 4\sqrt{12} \times \frac{\sqrt{3}}{2} = 16 - 12 = 4$

$z = \sqrt{4} =$ **2 mm**

5 $BC^2 = 12^2 + 37^2 - (2 \times 12 \times 37 \times \cos 45°)$

$BC^2 = 885.08...$

$BC = \sqrt{885.08...} =$ **29.8 miles** (3 s.f.)

6 $APB = 108° - 25° = 83°$

$AB^2 = 2^2 + 3^2 - (2 \times 2 \times 3 \times \cos 83°) = 11.5...$

$AB = \sqrt{11.5...} =$ **3.40 km** (3 s.f.)

Exercise 3

1 **a)** Area $= \frac{1}{2} \times 6 \times 5 \times \sin 104°$

$=$ **14.6 cm²** (3 s.f.)

b) Area $= \frac{1}{2} \times 8 \times 10 \times \sin 23°$

$=$ **15.6 m²** (3 s.f.)

Using the same method for c)-f):

c) **38.2 cm²** (3 s.f.) **d)** **123 mm²** (3 s.f.)

e) **43.7 in²** (3 s.f.) **f)** **16.7 cm²** (3 s.f.)

2 Area of sector $= \frac{67°}{360°} \times \pi \times 4.5^2 = 11.8...$ cm²

Area of triangle $= \frac{1}{2} \times 4.5 \times 4.5 \times \sin 67°$
$= 9.32...$ cm²

Area of sector $= 11.8... - 9.32...$

$=$ **2.52 cm²** (3 s.f.)

3 An equilateral triangle has sides of equal length and angles of equal size (60°).

Area $= \frac{1}{2} \times 32 \times 32 \times \sin 60° =$ **443 m²** (3 s.f.)

4 $\cos x = \frac{9^2 + 15^2 - 13^2}{2 \times 9 \times 15}$

$x = \cos^{-1}\left(\frac{9^2 + 15^2 - 13^2}{2 \times 9 \times 15}\right) =$ **59.5°** (3 s.f.)

Area $= \frac{1}{2} \times 9 \times 15 \times \sin 59.50...°$

$=$ **58.2 m²** (3 s.f.)

5 $\frac{10}{\sin 84°} = \frac{y}{\sin 49°}$

$y = \frac{10\sin 49°}{\sin 84°} = 7.5886... =$ **7.59 cm** (3 s.f.)

Missing angle $= 180° - 84° - 49° = 47°$

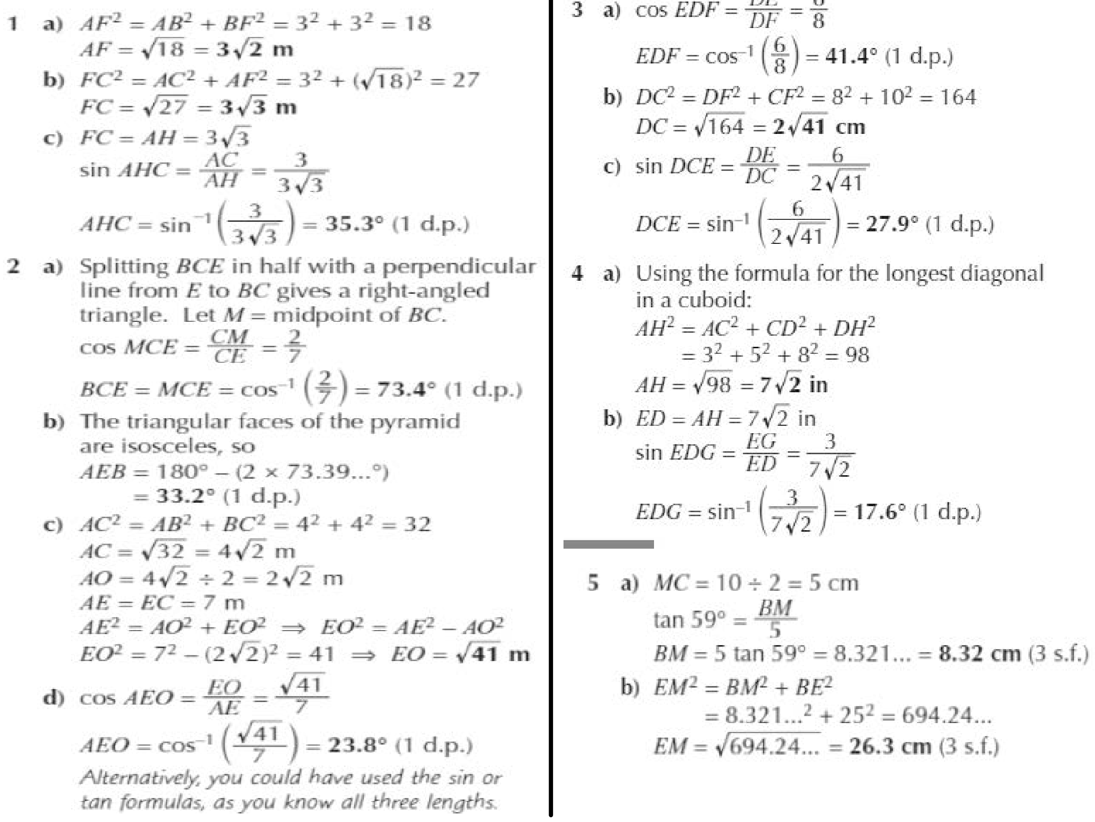
Area $= \frac{1}{2} \times 10 \times 7.5886... \times \sin 47°$

$=$ **27.7 cm²** (3 s.f.)

Model solutions on

5. Trigonometry in 3D

Exercise 1



1 **a)** $AF^2 = AB^2 + BF^2 = 3^2 + 3^2 = 18$

$AF = \sqrt{18} = \mathbf{3\sqrt{2}\ m}$

b) $FC^2 = AC^2 + AF^2 = 3^2 + (\sqrt{18})^2 = 27$

$FC = \sqrt{27} = \mathbf{3\sqrt{3}\ m}$

c) $FC = AH = 3\sqrt{3}$

$\sin AHC = \frac{AC}{AH} = \frac{3}{3\sqrt{3}}$

$AHC = \sin^{-1}\left(\frac{3}{3\sqrt{3}}\right) = \mathbf{35.3°}$ (1 d.p.)

2 **a)** Splitting BCE in half with a perpendicular line from E to BC gives a right-angled triangle. Let M = midpoint of BC.

$\cos MCE = \frac{CM}{CE} = \frac{2}{7}$

$BCE = MCE = \cos^{-1}\left(\frac{2}{7}\right) = \mathbf{73.4°}$ (1 d.p.)

b) The triangular faces of the pyramid are isosceles, so

$AEB = 180° - (2 \times 73.39...°)$

$= \mathbf{33.2°}$ (1 d.p.)

c) $AC^2 = AB^2 + BC^2 = 4^2 + 4^2 = 32$

$AC = \sqrt{32} = 4\sqrt{2}$ m

$AO = 4\sqrt{2} \div 2 = 2\sqrt{2}$ m

$AE = EC = 7$ m

$AE^2 = AO^2 + EO^2 \Rightarrow EO^2 = AE^2 - AO^2$

$EO^2 = 7^2 - (2\sqrt{2})^2 = 41 \Rightarrow EO = \mathbf{\sqrt{41}\ m}$

d) $\cos AEO = \frac{EO}{AE} = \frac{\sqrt{41}}{7}$

$AEO = \cos^{-1}\left(\frac{\sqrt{41}}{7}\right) = \mathbf{23.8°}$ (1 d.p.)

Alternatively, you could have used the sin or tan formulas, as you know all three lengths.

3 **a)** $\cos EDF = \frac{DE}{DF} = \frac{6}{8}$

$EDF = \cos^{-1}\left(\frac{6}{8}\right) = \mathbf{41.4°}$ (1 d.p.)

b) $DC^2 = DF^2 + CF^2 = 8^2 + 10^2 = 164$

$DC = \sqrt{164} = \mathbf{2\sqrt{41}\ cm}$

c) $\sin DCE = \frac{DE}{DC} = \frac{6}{2\sqrt{41}}$

$DCE = \sin^{-1}\left(\frac{6}{2\sqrt{41}}\right) = \mathbf{27.9°}$ (1 d.p.)

4 **a)** Using the formula for the longest diagonal in a cuboid:

$AH^2 = AC^2 + CD^2 + DH^2$

$= 3^2 + 5^2 + 8^2 = 98$

$AH = \sqrt{98} = \mathbf{7\sqrt{2}\ in}$

b) $ED = AH = 7\sqrt{2}$ in

$\sin EDG = \frac{EG}{ED} = \frac{3}{7\sqrt{2}}$

$EDG = \sin^{-1}\left(\frac{3}{7\sqrt{2}}\right) = \mathbf{17.6°}$ (1 d.p.)

5 **a)** $MC = 10 \div 2 = 5$ cm

$\tan 59° = \frac{BM}{5}$

$BM = 5 \tan 59° = 8.321... = \mathbf{8.32\ cm}$ (3 s.f.)

b) $EM^2 = BM^2 + BE^2$

$= 8.321...^2 + 25^2 = 694.24...$

$EM = \sqrt{694.24...} = \mathbf{26.3\ cm}$ (3 s.f.)

Exercise 2

1 **a)** $\cos JIK = \dfrac{IJ^2 + IK^2 - JK^2}{2 \times IJ \times IK} = \dfrac{5^2 + 11^2 - 8^2}{2 \times 5 \times 11}$

$JIK = \cos^{-1}\left(\dfrac{5^2 + 11^2 - 8^2}{2 \times 5 \times 11}\right) = \mathbf{41.8°}$ (1 d.p.)

b) Area $= \dfrac{1}{2} \times 5 \times 11 \times \sin 41.8...° = \mathbf{18.3\ m^2}$

c) Volume = 18.3... × 9

= **165 m^3** (to nearest m^3)

2 **a)** $CE^2 = AC^2 + AE^2 = 4^2 + 6^2 = 52$

$CE = \sqrt{52} = \mathbf{2\sqrt{13}\ m}$

b) $CH^2 = CD^2 + DH^2 = 8^2 + 6^2 = 100$

$CH = \sqrt{100} = \mathbf{10\ m}$

c) $EH^2 = EF^2 + FH^2 = 8^2 + 4^2 = 80$

$EH = \sqrt{80} = \mathbf{4\sqrt{5}\ m}$

d) $\cos ECH = \dfrac{CE^2 + CH^2 - EH^2}{2 \times CE \times CH}$

$= \dfrac{52 + 100 - 80}{2 \times 2\sqrt{13} \times 10}$

$ECH = \cos^{-1}\left(\dfrac{52 + 100 - 80}{2 \times 2\sqrt{13} \times 10}\right)$

$= \mathbf{60.1°}$ (1 d.p.)

3 **a)** $PU^2 = PQ^2 + QU^2 = 12^2 + 10^2 = 244$

$PU = \sqrt{244} = 2\sqrt{61}$

$PS^2 = PR^2 + RS^2 = 5^2 + 12^2 = 169$

$PS = \sqrt{169} = 13$ m

$SU^2 = SW^2 + UW^2 = 10^2 + 5^2 = 125$

$SU = \sqrt{125} = 5\sqrt{5}$ m

$\cos PSU = \dfrac{PS^2 + SU^2 - PU^2}{2 \times PS \times SU}$

$= \dfrac{169 + 125 - 244}{2 \times 13 \times 5\sqrt{5}}$

$PSU = \cos^{-1}\left(\dfrac{169 + 125 - 244}{2 \times 13 \times 5\sqrt{5}}\right)$

$= \mathbf{80.1°}$ (1 d.p.)

b) Area $= \dfrac{1}{2} \times 13 \times 5\sqrt{5} \times \sin 80.09...°$

$= \mathbf{71.6\ m^2}$ (1 d.p.)